

# 変位電流を含めた有限要素法 周波数領域の電磁場解析

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## 周波数領域有限要素法の利点

- メッシュの自由度が大きい。
- 従来の有限要素法手法やツールが使用可能。
- 定常問題が直接的に解ける。1周波数の解析なら、FDTDより高速？

# 解析手法

- 変位電流を含んだFull Maxwell方程式をA- $\phi$ 法,  $j\omega$ 法で解く
- ゲージ条件は入れない
- 複素ICCG法
- 完全導体壁や表面インピーダンス壁
- デルタギャップ給電, 導波モード印可, インピーダンスマッチング
- 本報告は閉領域解析のみ

# 解析例

- 導波路, 共鳴空洞

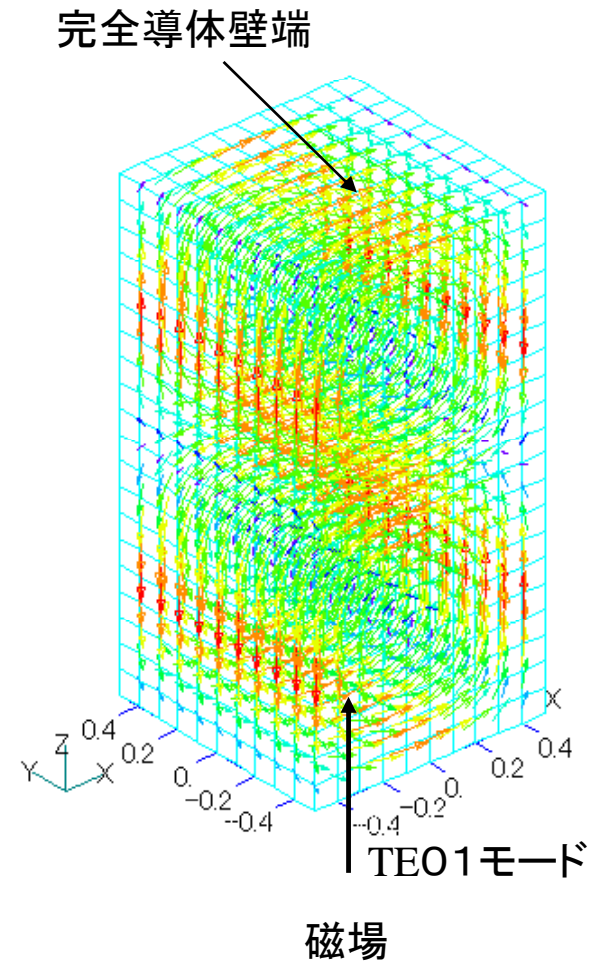
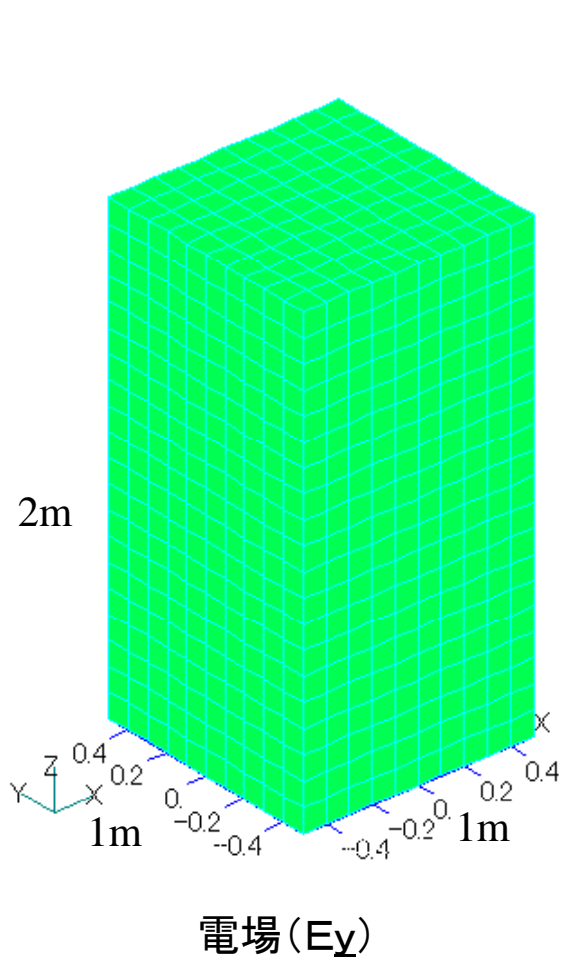
共鳴周波数, インピーダンス, Sパラメータ, Q値

- 低周波電場

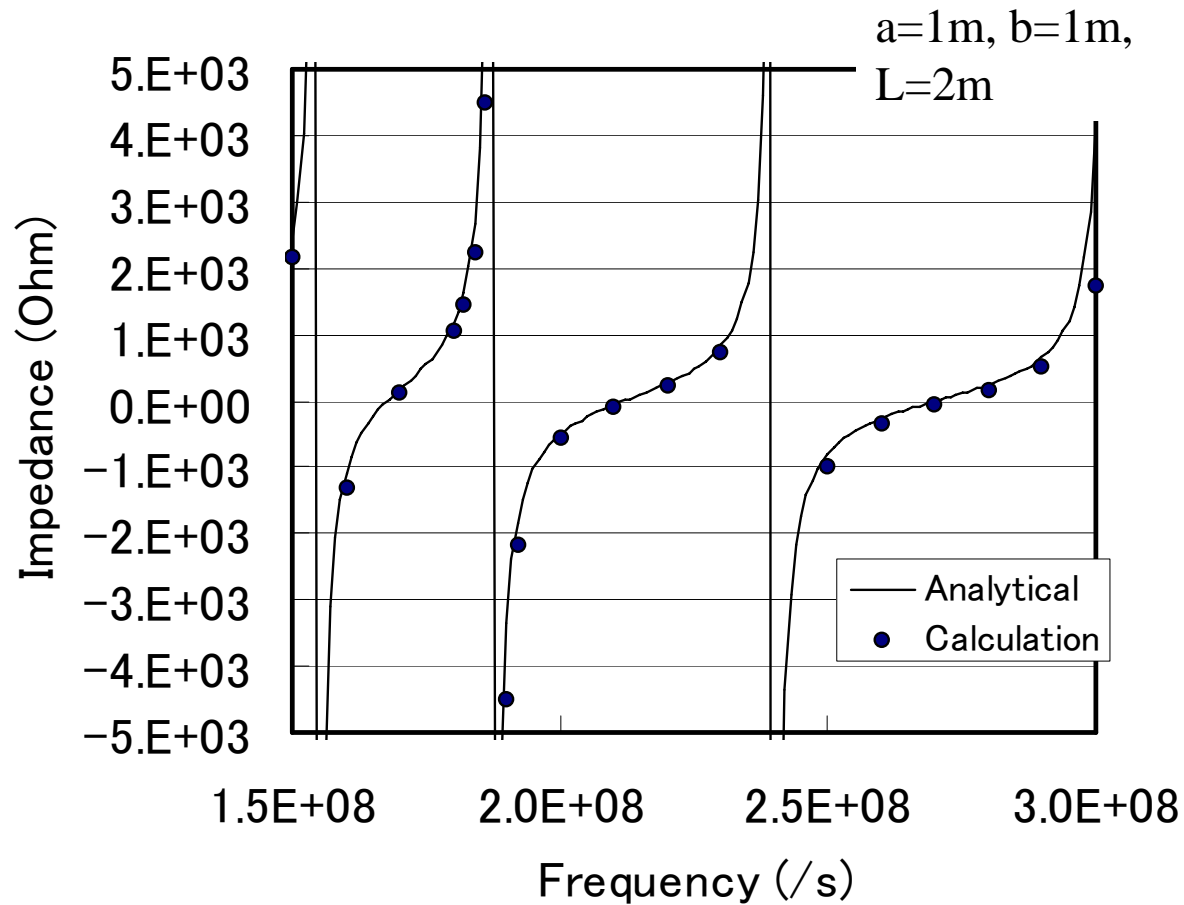
- アンテナ

- 解析解や計測値との比較

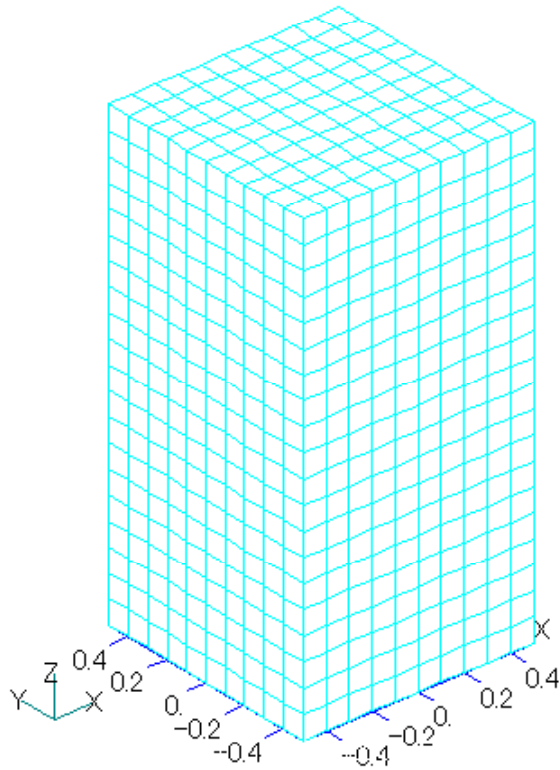
# 導波管定在波 (TE<sub>10</sub>モード, 210MHz)



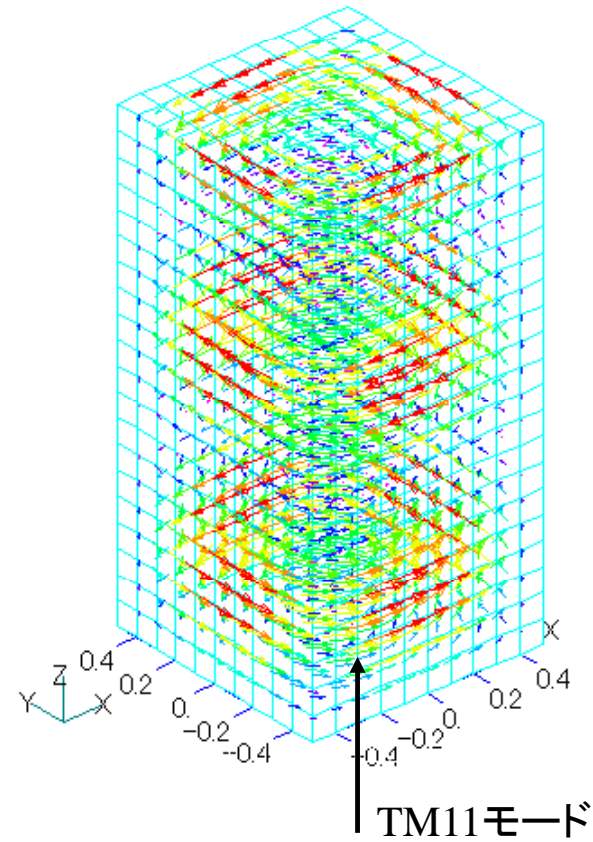
# 導波管定在波インピーダンス (TE<sub>10</sub>モード)



# 導波管定在波 (TM<sub>11</sub>モード, 280MHz)



電場



磁場

# 矩形導波管固有モード

## TE<sub>mn</sub>モード

## TM<sub>mn</sub>モード

特性界インピーダンス  $Z_m = \frac{\mu\omega}{k_z}$

$Z_e = \frac{\epsilon k_z}{\omega}$

波数

$$k_z^2 = k^2 - k_t^2 = k^2 - \left(\frac{m\pi}{a}\right)^2 - \left(\frac{n\pi}{b}\right)^2$$

モード

$$e_x = \frac{n\pi}{b} Z_m \cdot \cos \frac{m\pi}{a} x \cdot \sin \frac{n\pi}{b} y$$

$$e_x = -\frac{m\pi}{a} \cos \frac{m\pi}{a} x \cdot \sin \frac{n\pi}{b} y$$

$$e_y = -\frac{m\pi}{a} Z_m \cdot \sin \frac{m\pi}{a} x \cdot \cos \frac{n\pi}{b} y$$

$$e_y = -\frac{n\pi}{b} \sin \frac{m\pi}{a} x \cdot \cos \frac{n\pi}{b} y$$

$$e_z = 0$$

$$e_z = \frac{k^2 - k_z^2}{jk_z} \sin \frac{m\pi}{a} x \cdot \sin \frac{n\pi}{b} y$$

$$h_x = \frac{m\pi}{a} \sin \frac{m\pi}{a} x \cdot \cos \frac{n\pi}{b} y$$

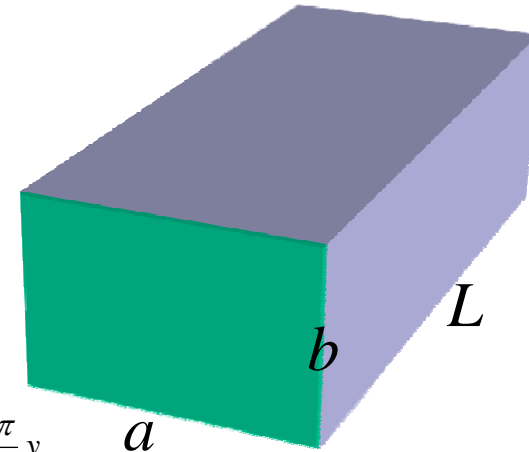
$$h_x = \frac{n\pi}{b Z_e} \sin \frac{m\pi}{a} x \cdot \cos \frac{n\pi}{b} y$$

$$h_y = \frac{n\pi}{b} \cdot \cos \frac{m\pi}{a} x \cdot \sin \frac{n\pi}{b} y$$

$$h_y = -\frac{m\pi}{a Z_e} \cdot \cos \frac{m\pi}{a} x \cdot \sin \frac{n\pi}{b} y$$

$$h_z = \frac{k^2 - k_z^2}{jk_z} \cos \frac{m\pi}{a} x \cdot \cos \frac{n\pi}{b} y$$

$$h_z = 0$$

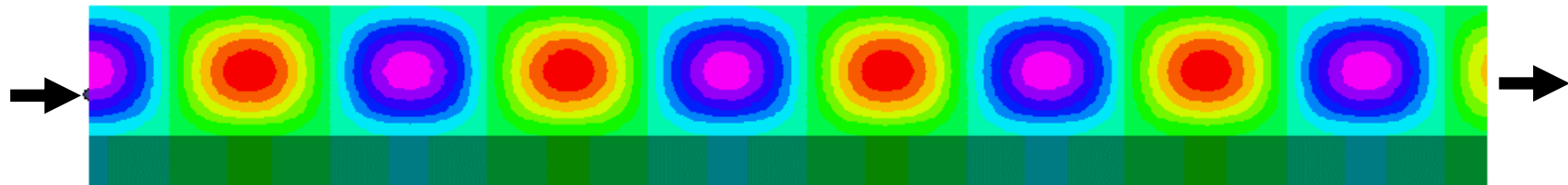


先端短絡インピーダンス

$$Z = jZ_0 \tan k_z L$$

# 矩形導波管TE10モード進行波

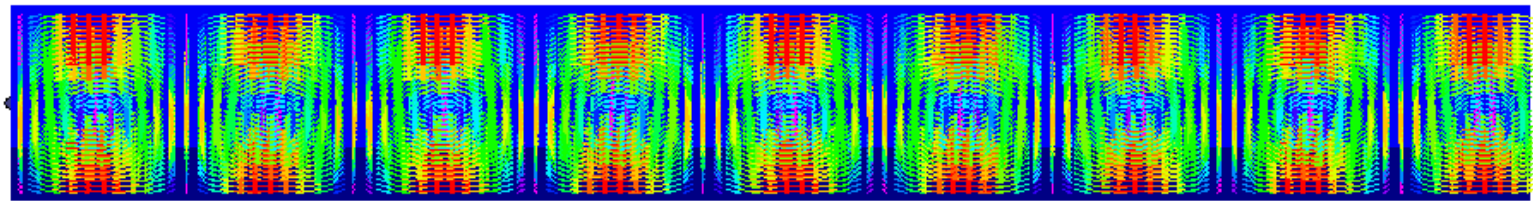
電場



H: given

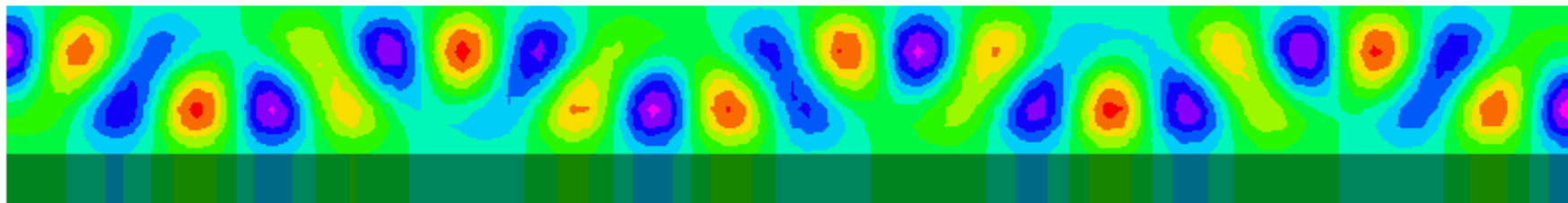
$$E = Z_0 H$$

磁場

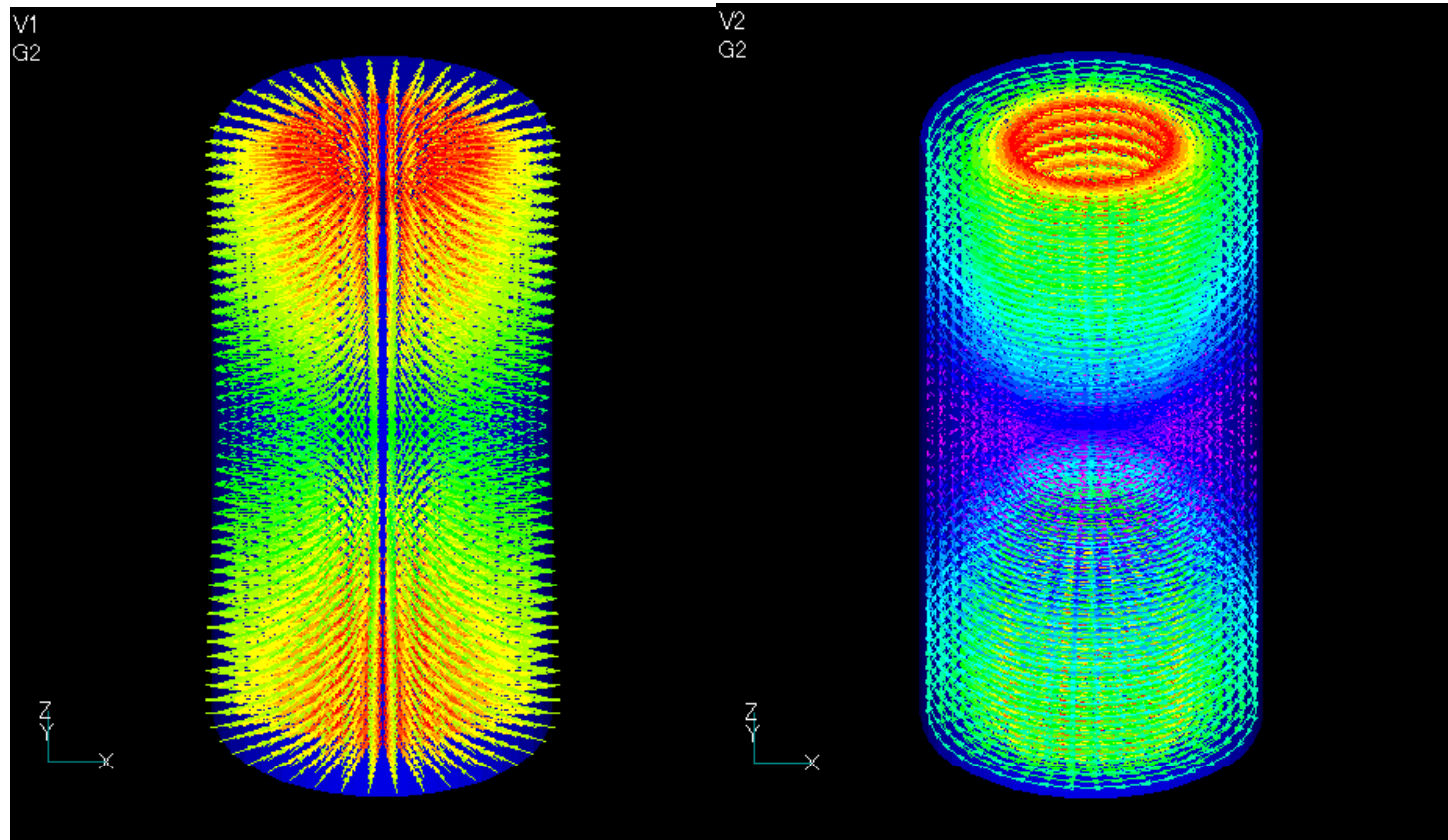


x<sup>-z</sup>

# 矩形導波管 TE10+TE20



# 同軸TEMモード



電場

磁場

# 同軸TEMモード 電送損失

## •解析解

$$\alpha_c = \frac{1}{4\pi Z} \left( \frac{R_{sa}}{a} + \frac{R_{sb}}{b} \right) = \frac{8 \times 10^{-3}}{4\pi \times 50} \left( \frac{1}{16.7 \times 10^{-3}} + \frac{1}{38.45 \times 10^{-3}} \right) = 1.094 \times 10^{-3} (Np/m)$$

## •計算結果

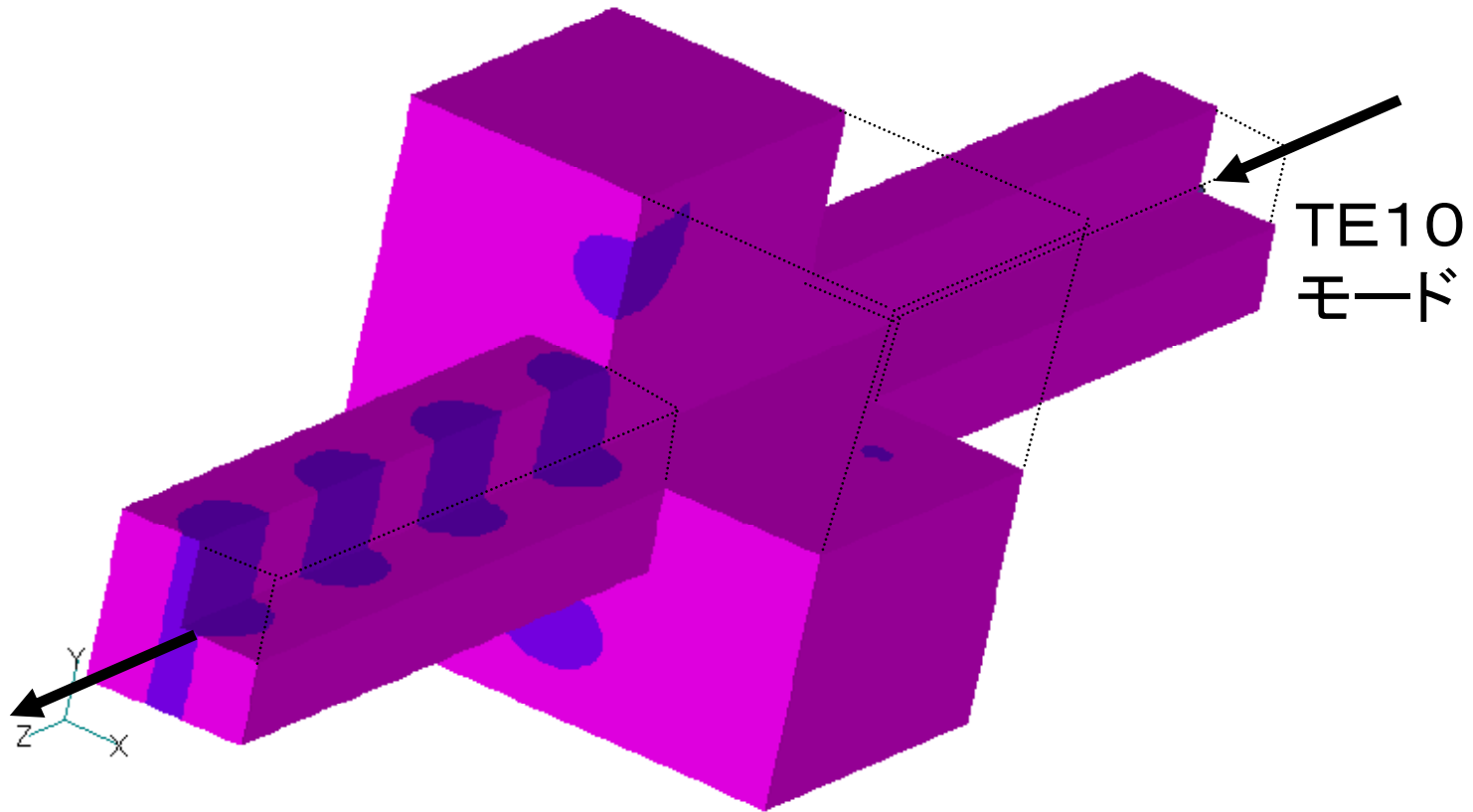
エネルギーより

$$\alpha_c = \frac{P_e}{2v_p (\tilde{W}_m + \tilde{W}_e)} = \frac{0.0619596}{2 \times 3 \times 10^8 (4.708 \times 10^{-8} + 4.708 \times 10^{-8})} = 1.097 \times 10^{-3} (Np/m)$$

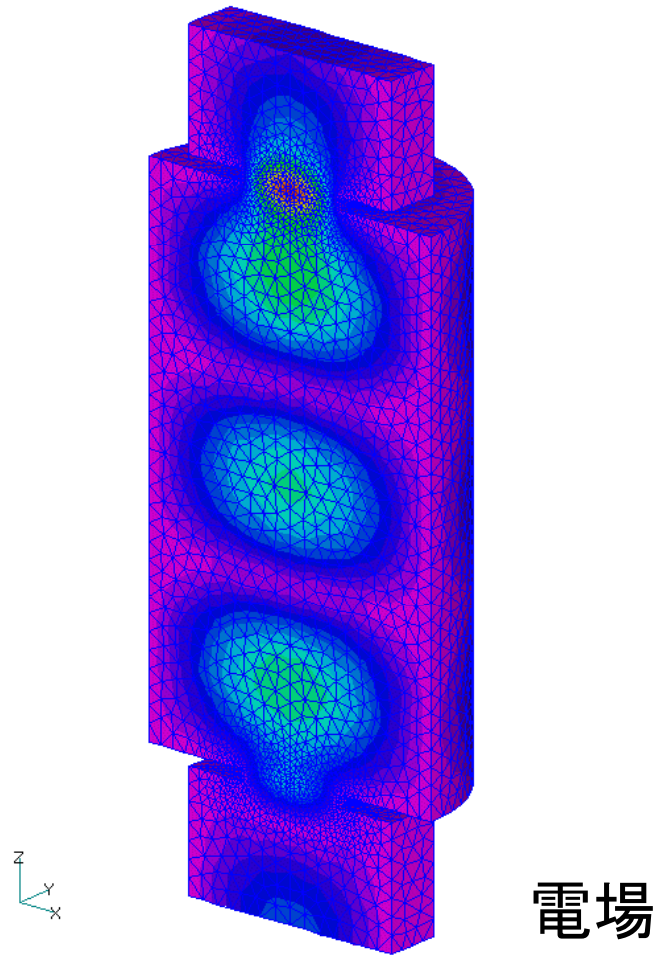
透過係数より

$$\alpha_c = \frac{1 - |S_{12}|^2}{2L} = \frac{1 - 0.999834^2}{2 \times 0.15} = 1.107 \times 10^{-3} (Np/m)$$

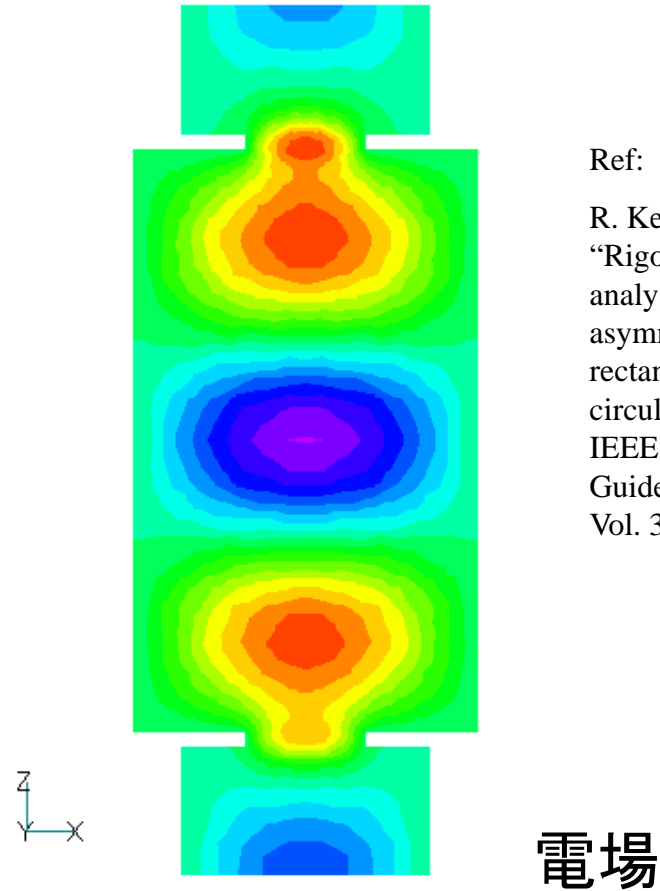
# 2端子回路



# 方形アイリス結合形導波管フィルター



11.79GHz (resonant)

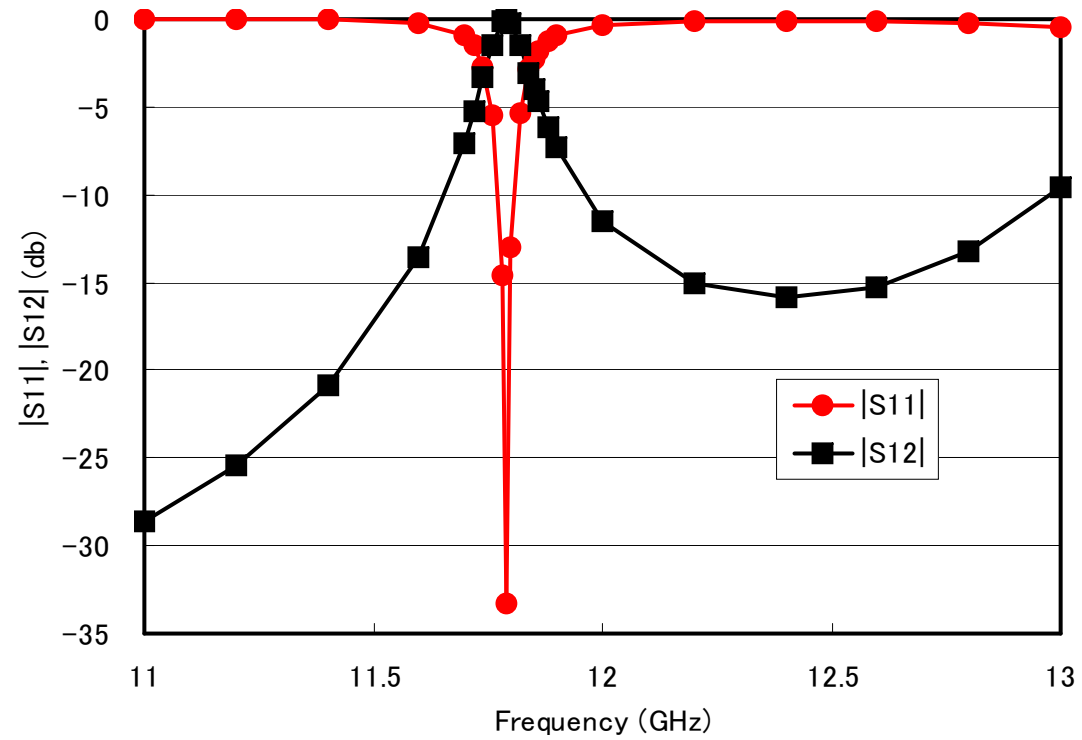


11.85GHz (off resonant)<sub>14</sub>

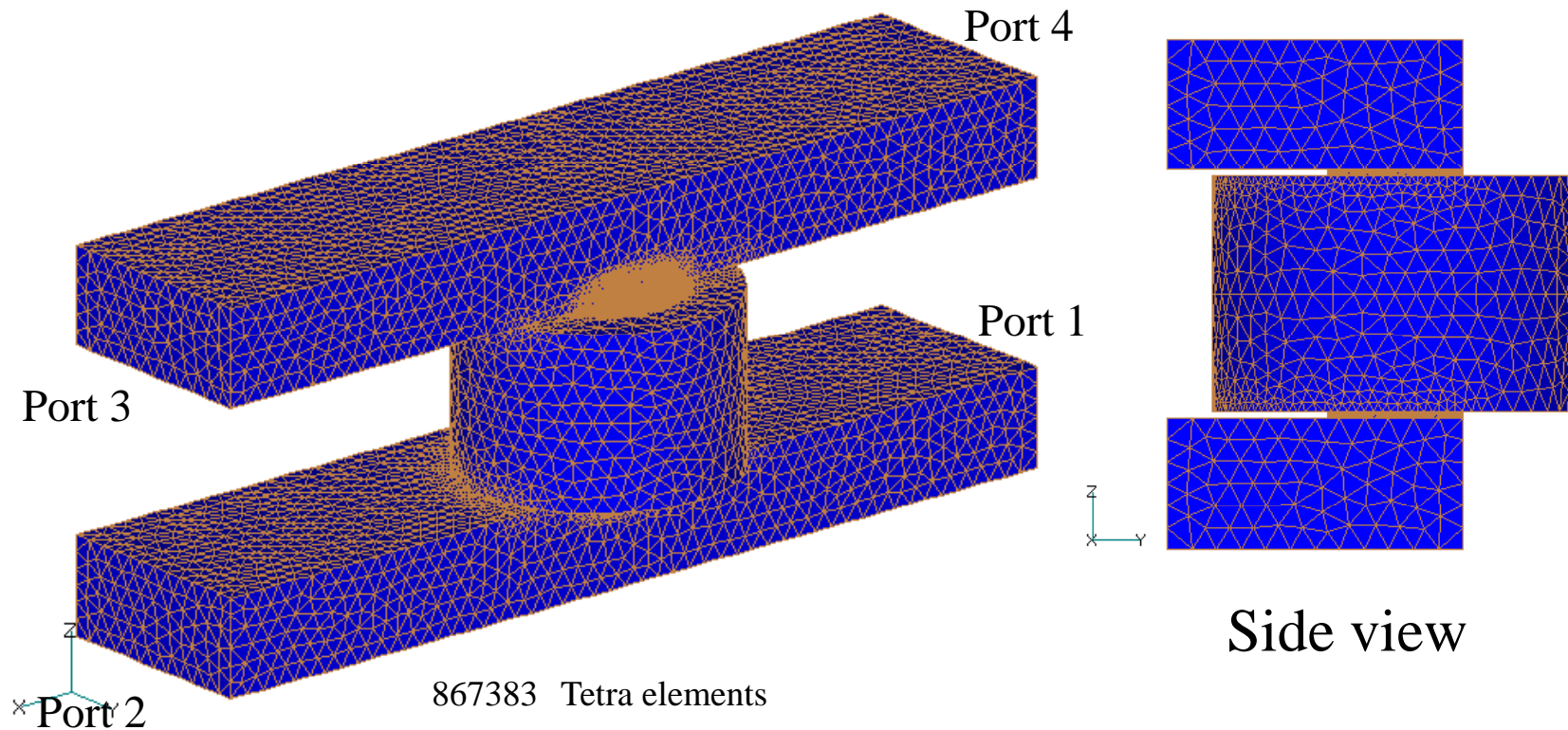
Ref:

R. Keller and F. Arbdt,  
“Rigorous model  
analysis of the  
asymmetric  
rectangular iris in  
circular waveguides”,  
IEEE Microwave and  
Guided Wave Letters,  
Vol. 3, No. 6, 1993.

# 透過・反射係数

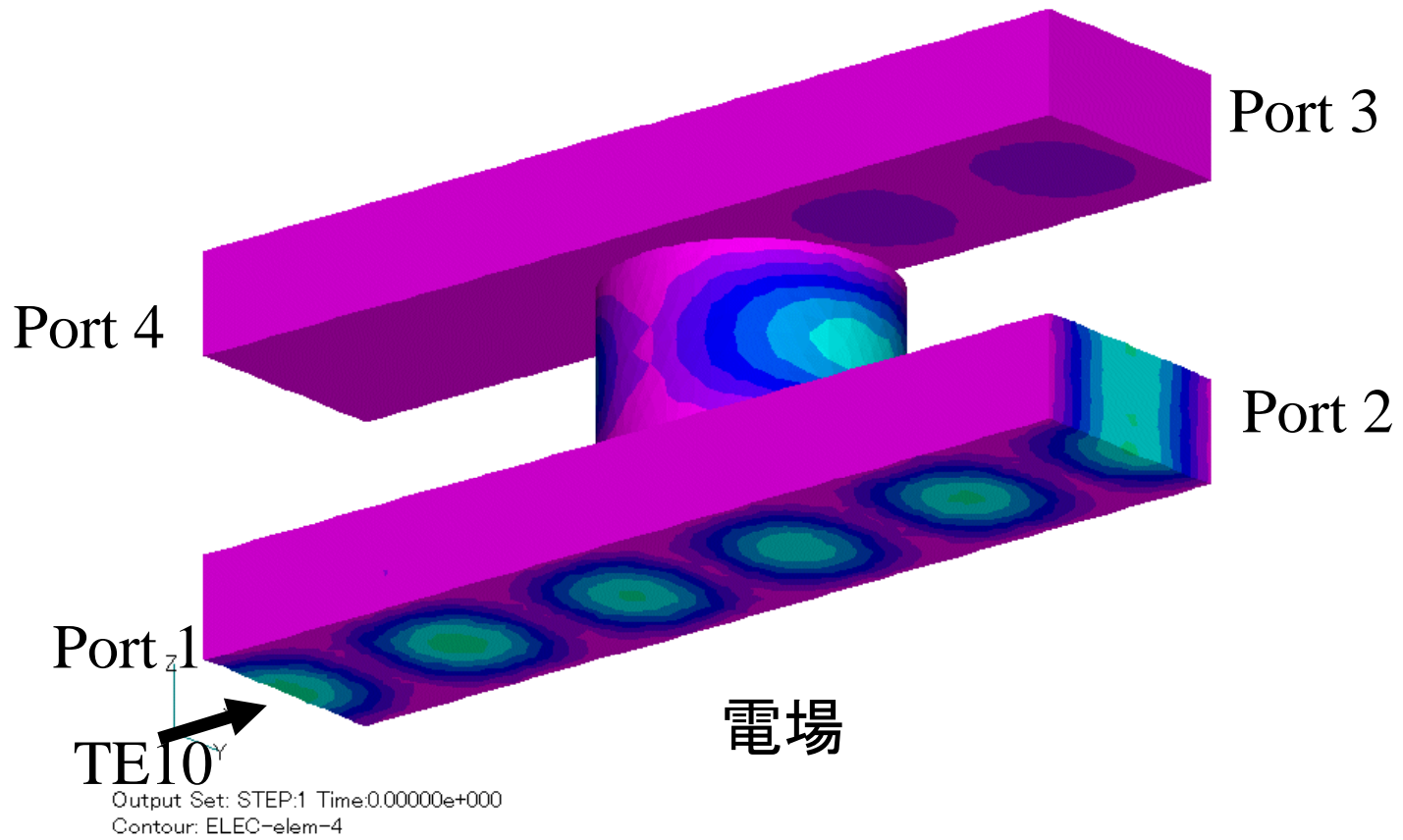


# Xバンド1段Directional Filter

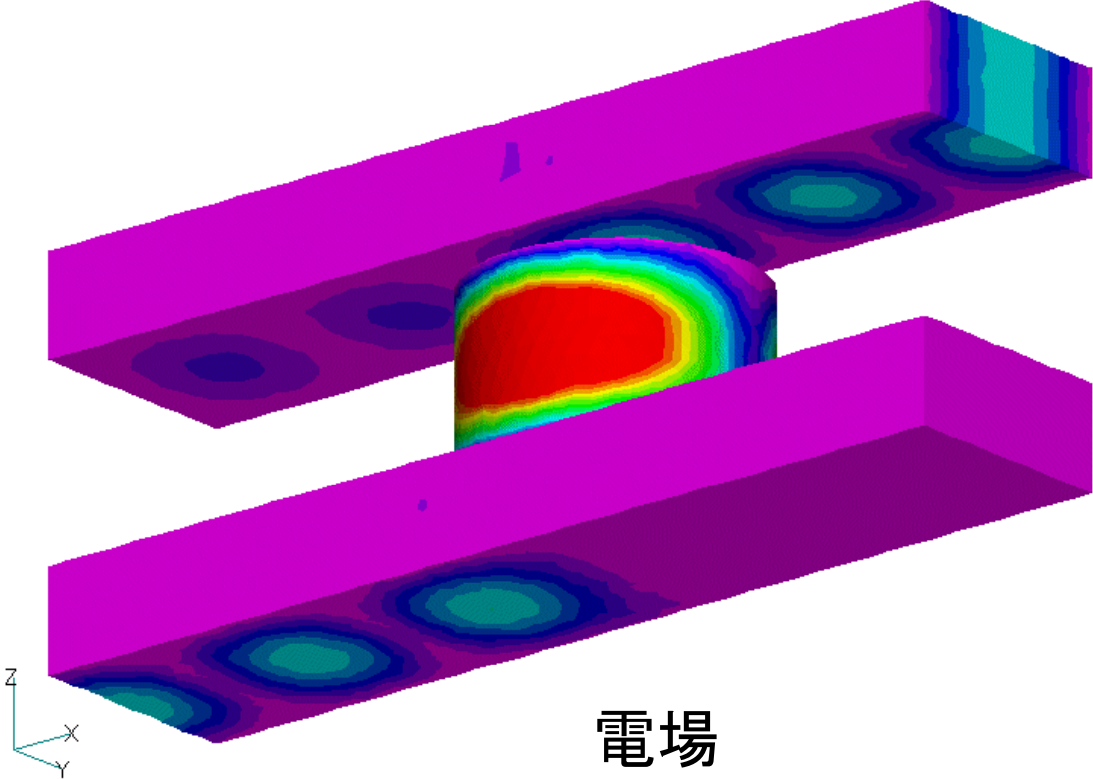


Ref:山下栄吉編著“電磁場問題解析の実際”、電子  
情報通信学会 (1993)

9.75GHz (off resonant)



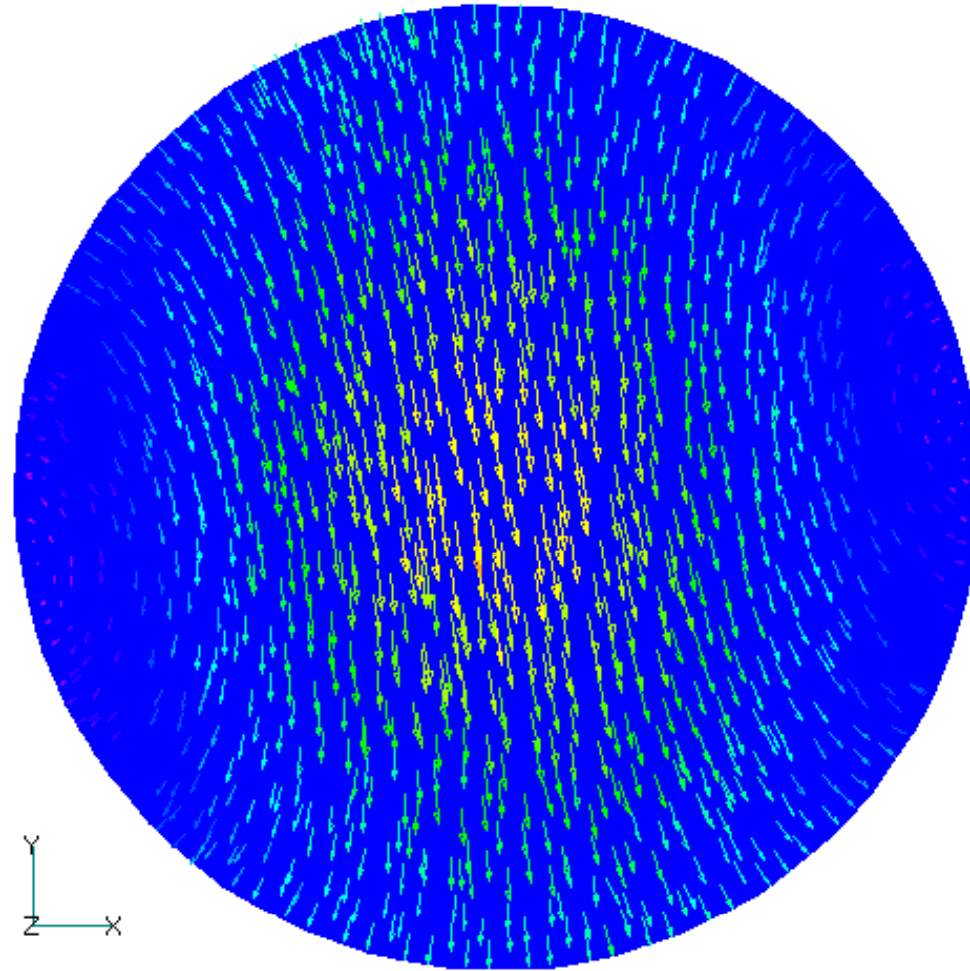
9.821GHz (resonant)



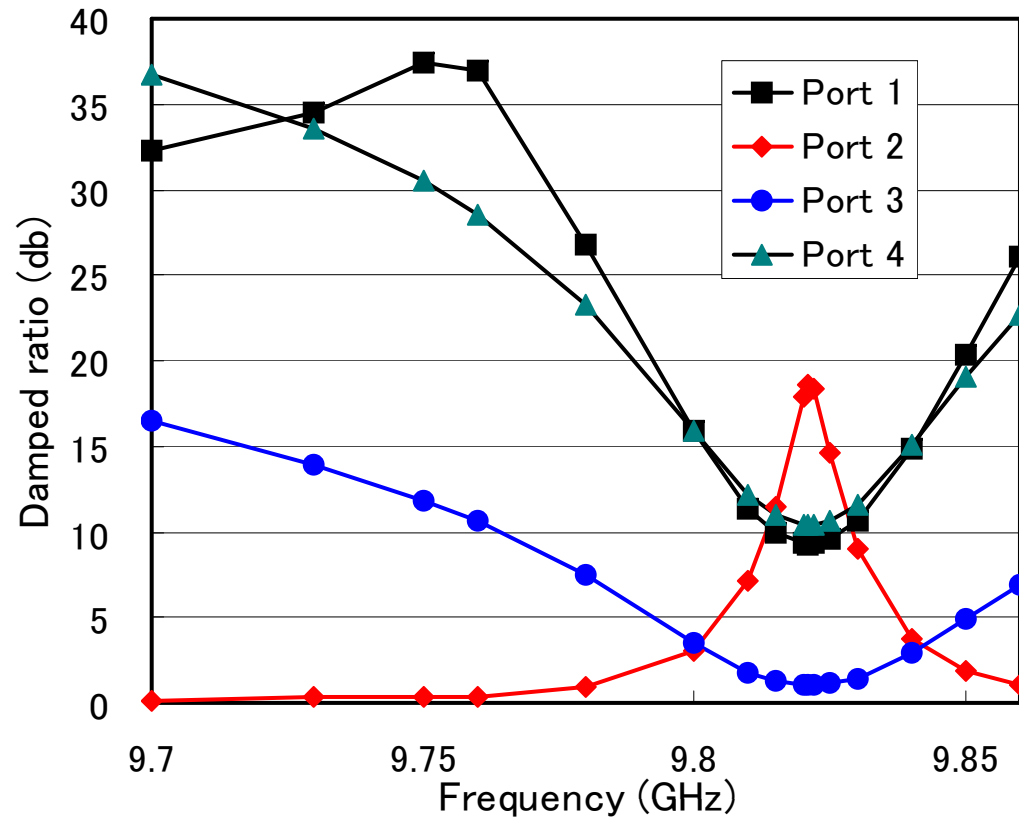
電場

Output Set: STEP:1 Time:0.00000e+000  
Contour: ELEC-elem-4

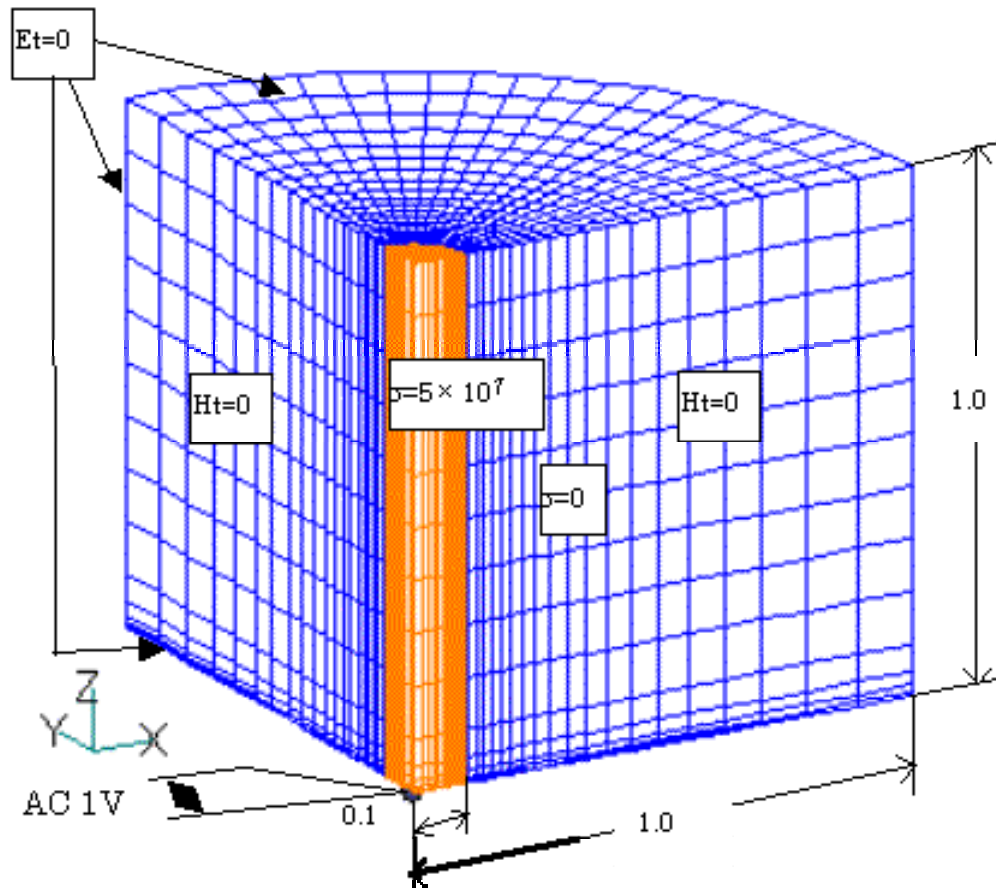
# 円筒空洞内の回転電場



# 減衰率周波数依存性

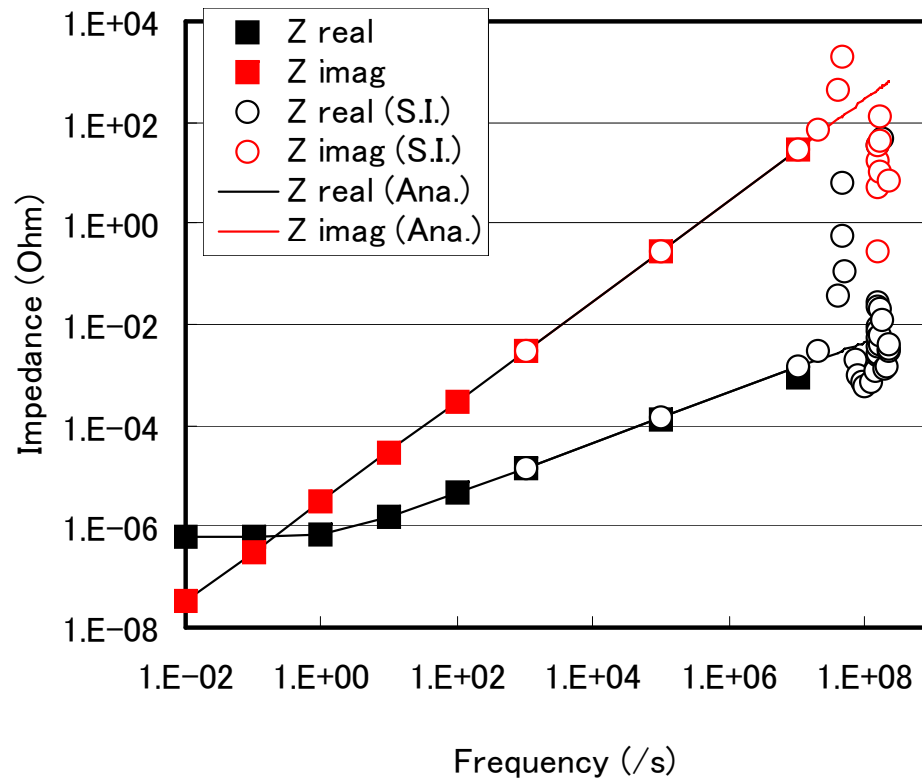


# 低周波から高周波まで 解析モデル



- 変位電流を全周波数領域で解析に含む
- 交流1Vを無限小厚のギャップ間に印可(デルタギャップ給電)
- インピーダンスをギャップを通過する電流より計算

# インピーダンス周波数依存性



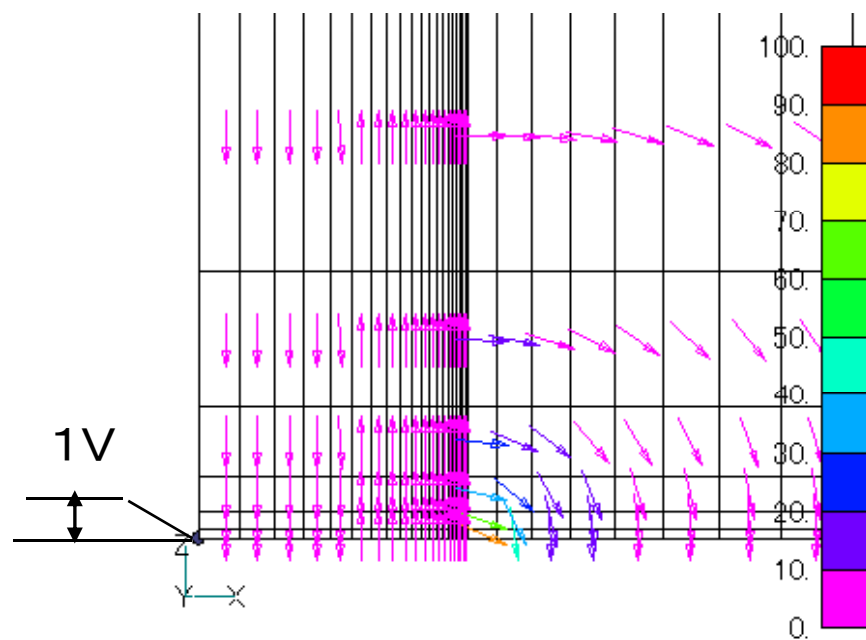
■導体をバルク要素で解析した場合, 高周波で特に抵抗(実数)分が解析値からはずれずれる。インダクタンス(虚数)分は, 空気領域が支配的であり, 差は小さい。

■表面インピーダンス壁(S.I.)はスキン厚が小さいとき, 良い近似となる。

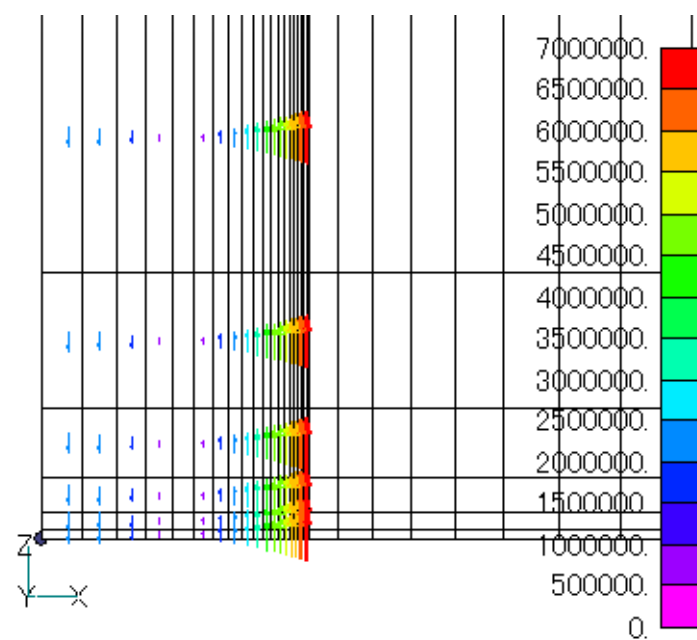
■波長が, 系の大きさに近づくと変位電流を無視した解析解(Ana. )より, 大きく離れてくる。

# 低周波(1Hz)での電場と電流

位相0度

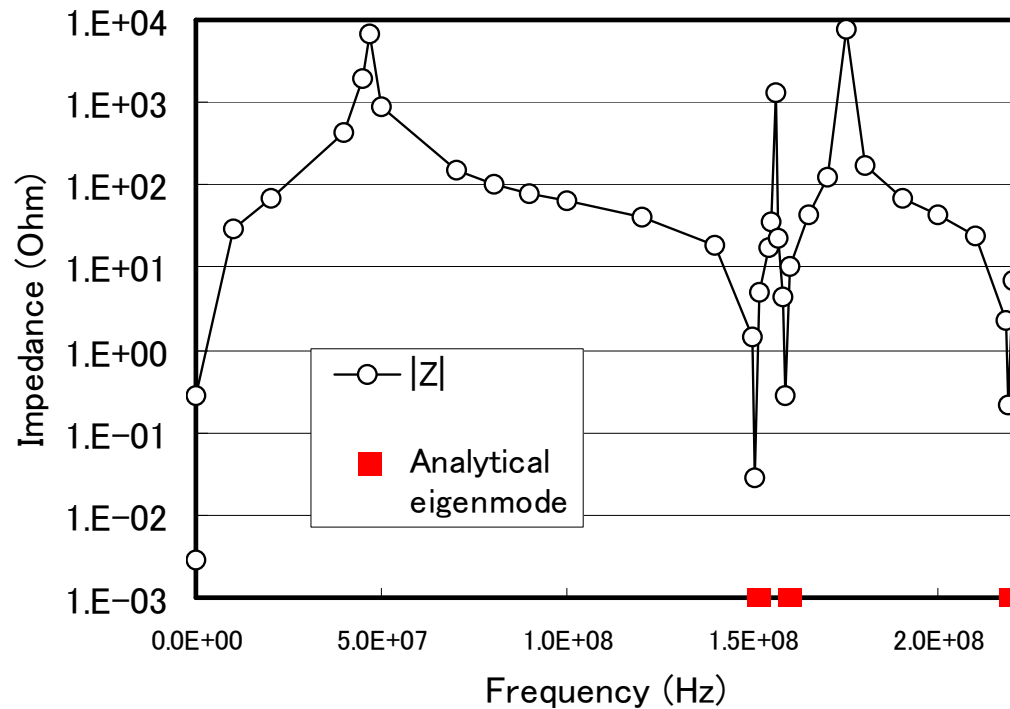


電場(E)分布



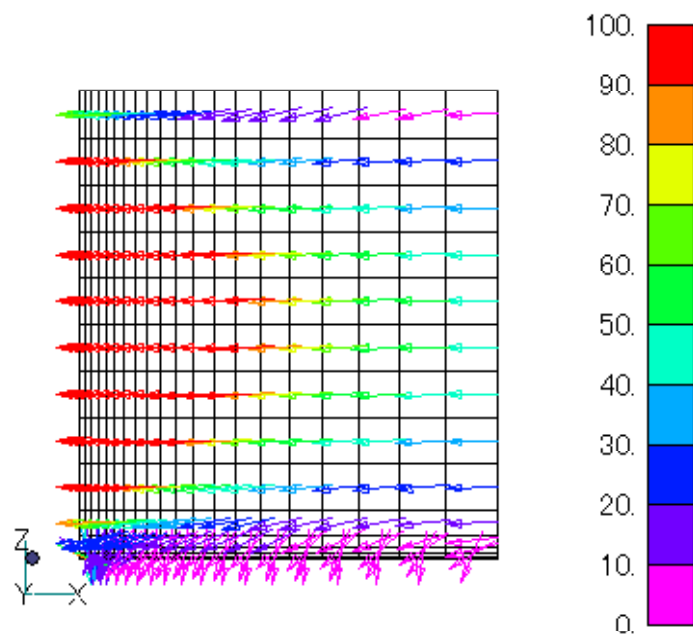
電流密度(J)分布

# インピーダンス周波数依存性(高周波領域)

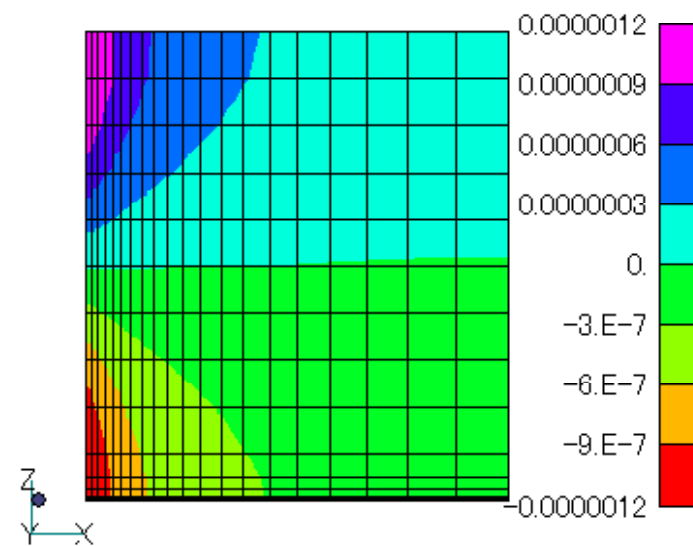


■高周波領域では共鳴現象が起こる。インピーダンスが極小になる周波数は、空洞共鳴周波数解析解と一致する。

# 共鳴点での電磁場の分布(151MHz)

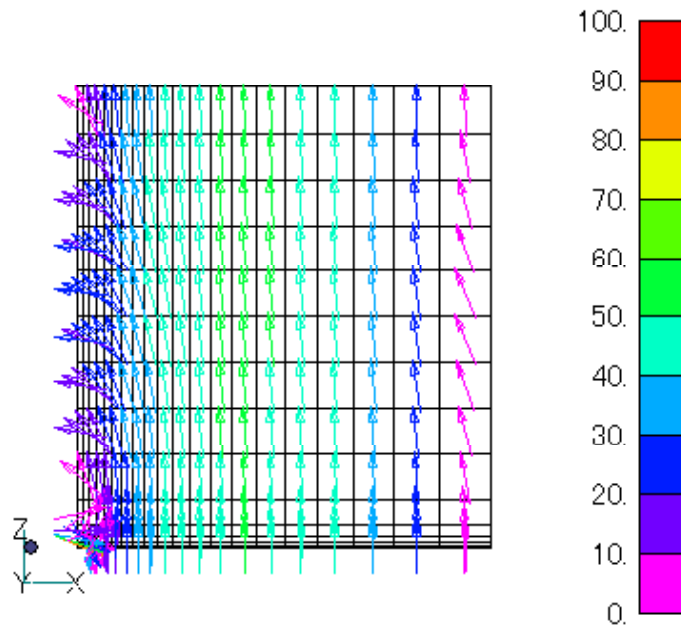


電場

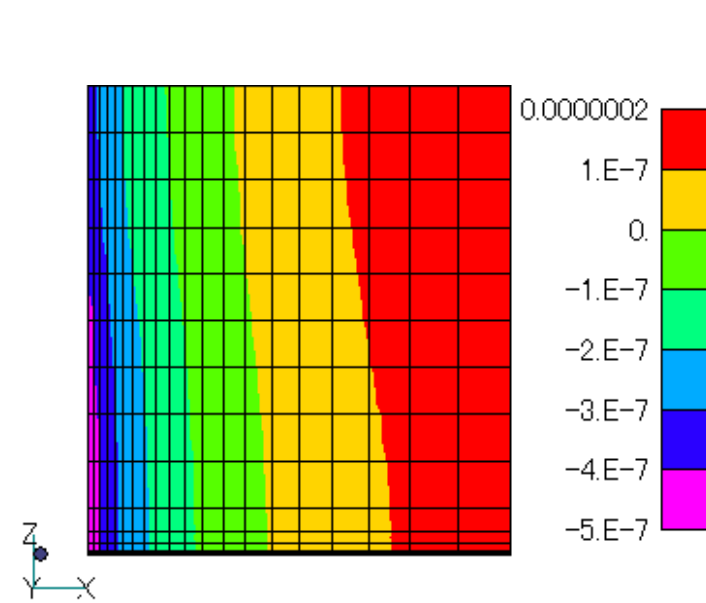


磁場(B $\theta$ )

# 共鳴点での電磁場の分布(159MHz)

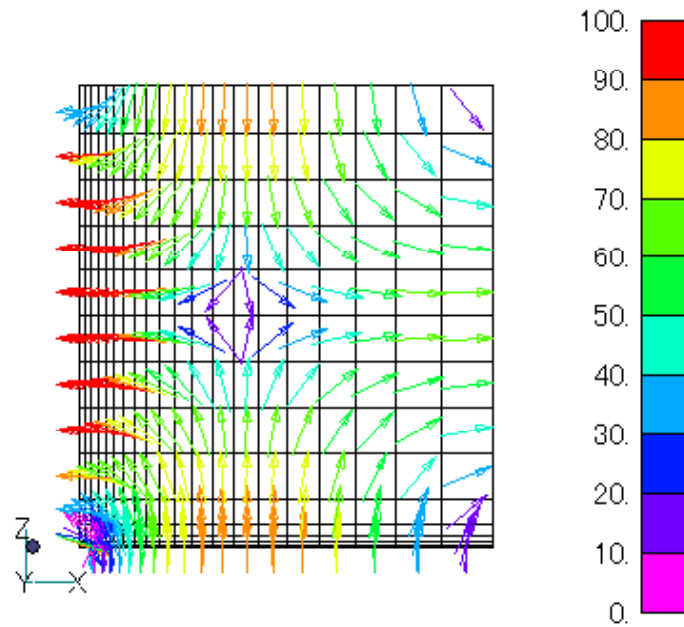


電場

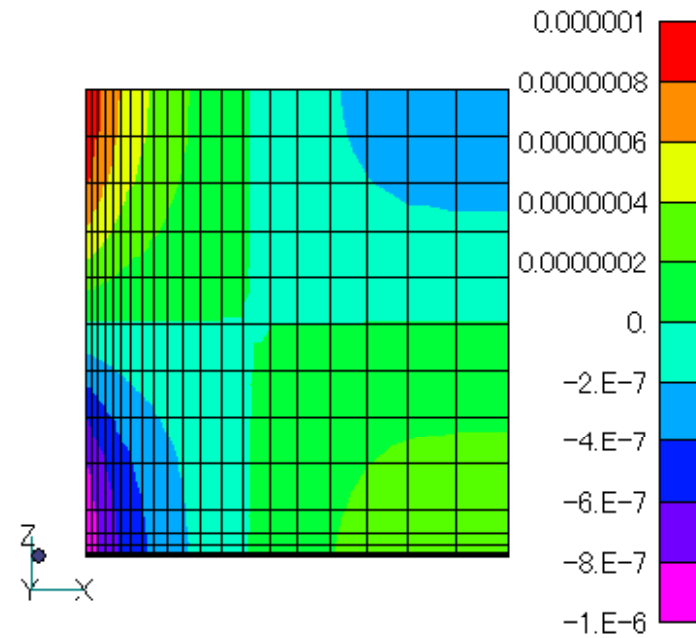


磁場(B0)

# 共鳴点での電磁場の分布(219MHz)

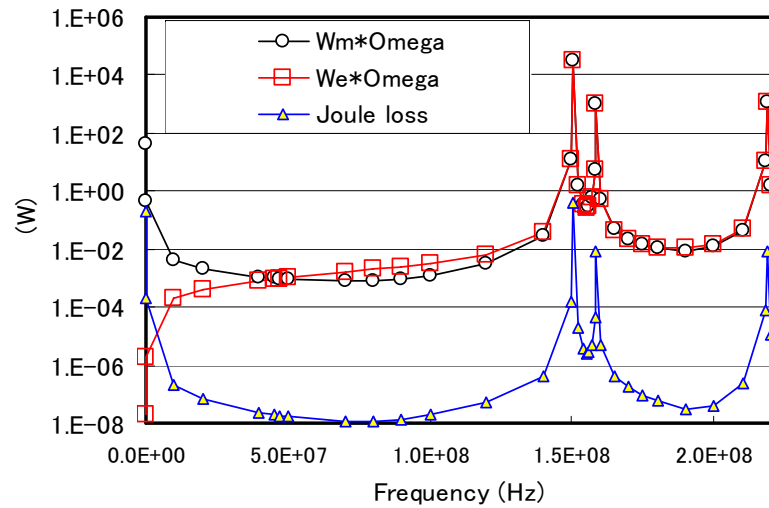


電場

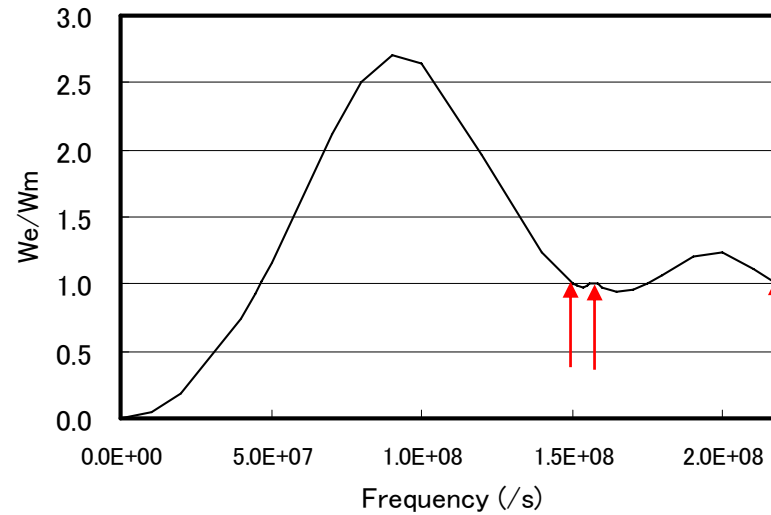


磁場(B $\theta$ )

# 共鳴点とエネルギーの関係



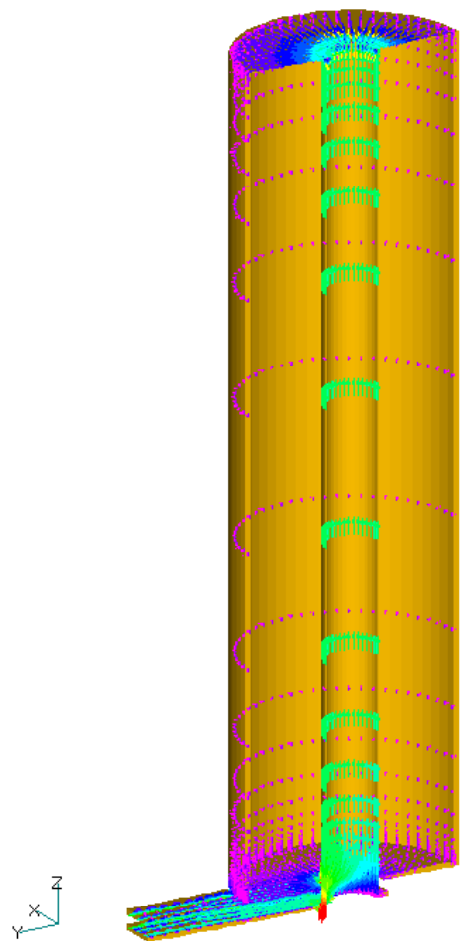
電磁場エネルギーとジュールロス  
(印可電圧1V)



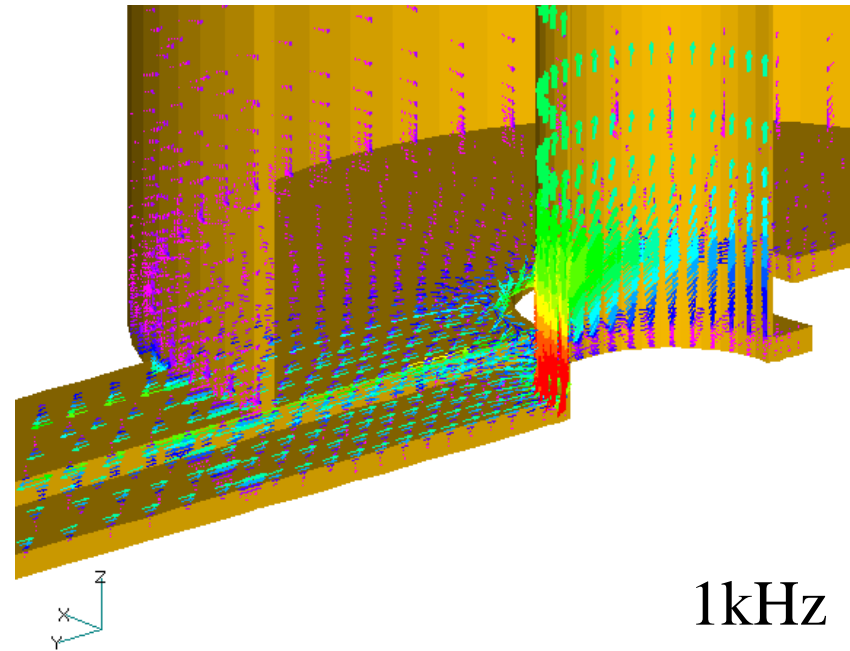
電磁場エネルギー比

■電気エネルギーの磁気エネルギーの比が減少し1に等しくなった点が共鳴点

# 低周波における渦電流と電場の同時解析

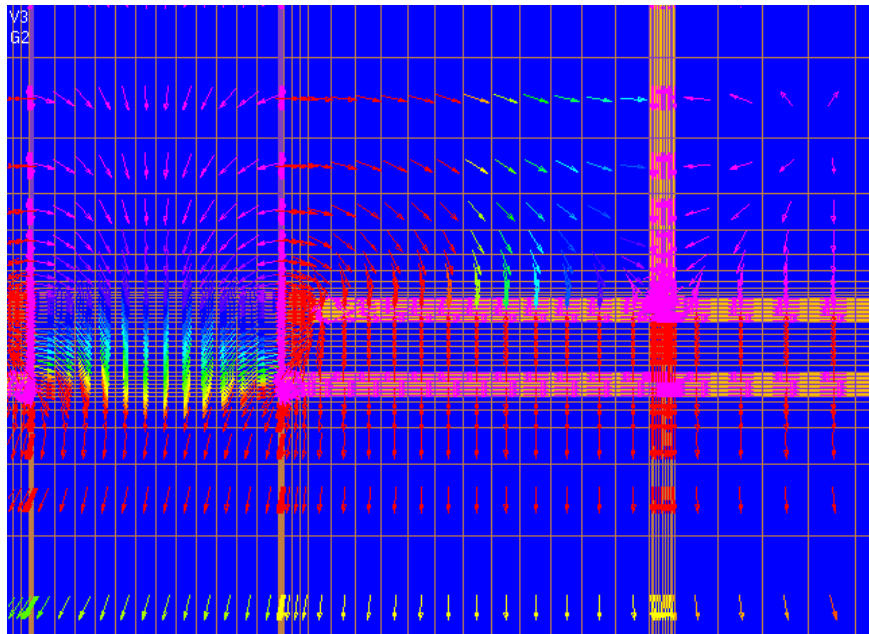
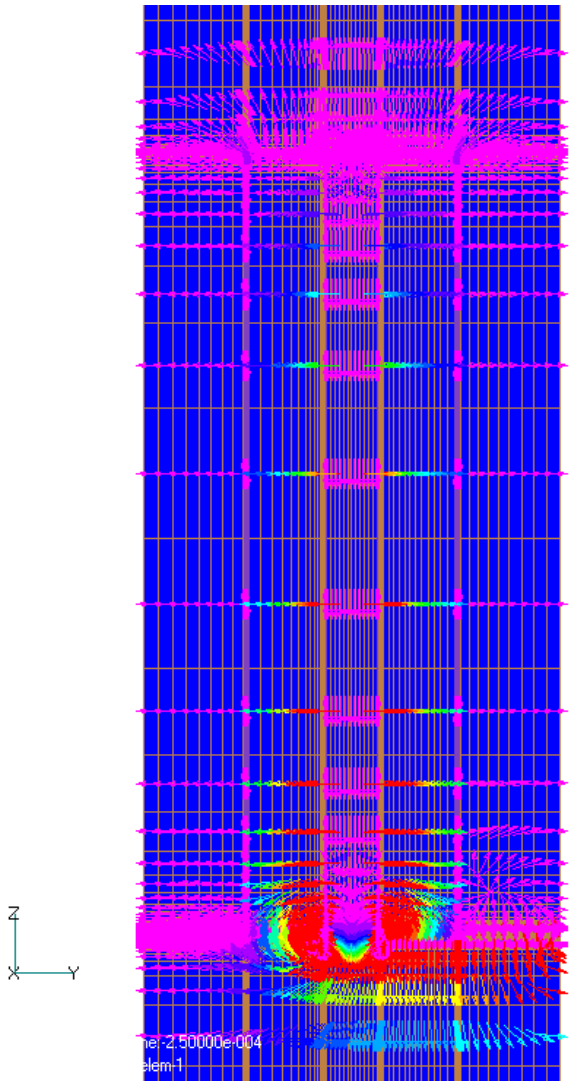


渦電流

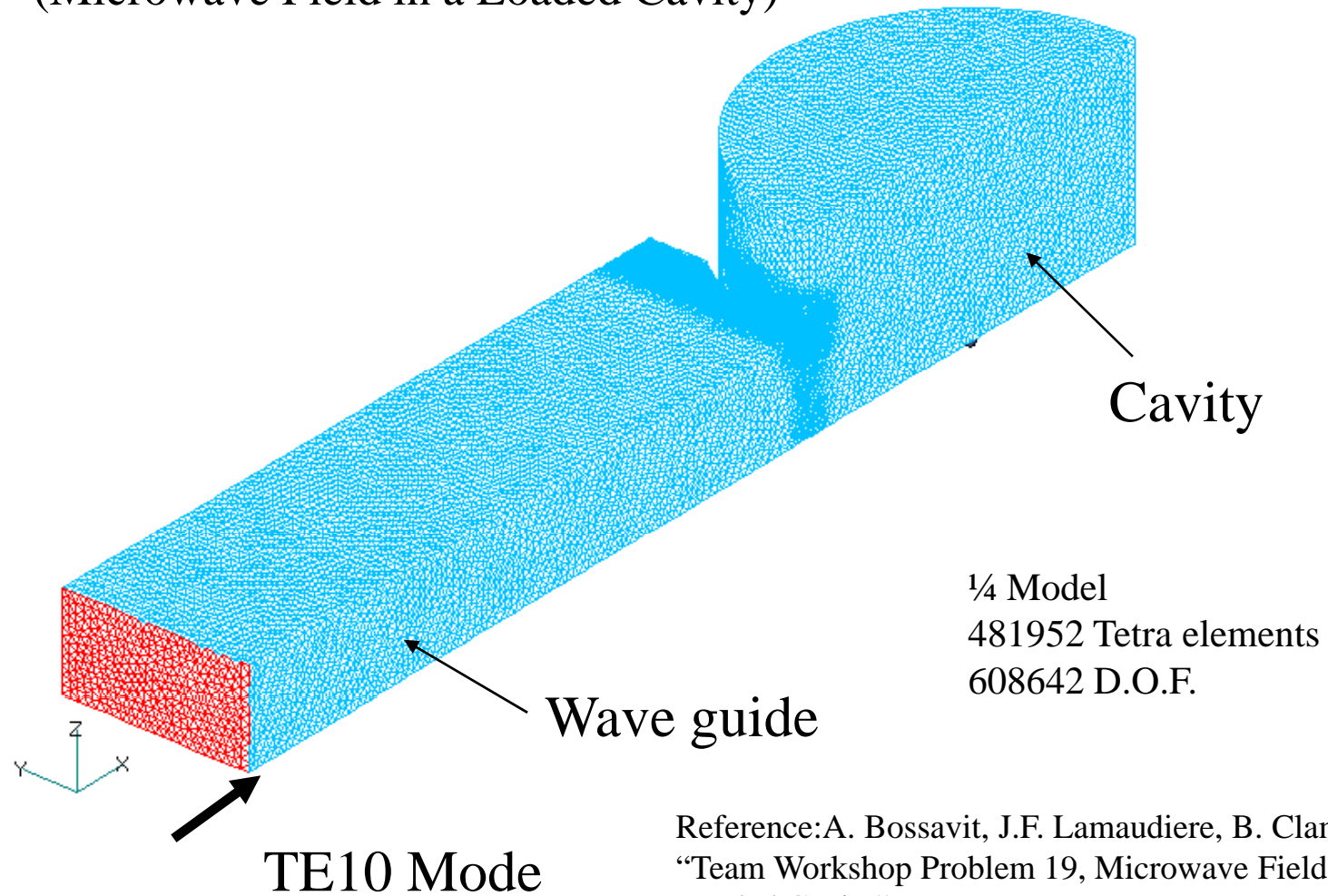


1kHz

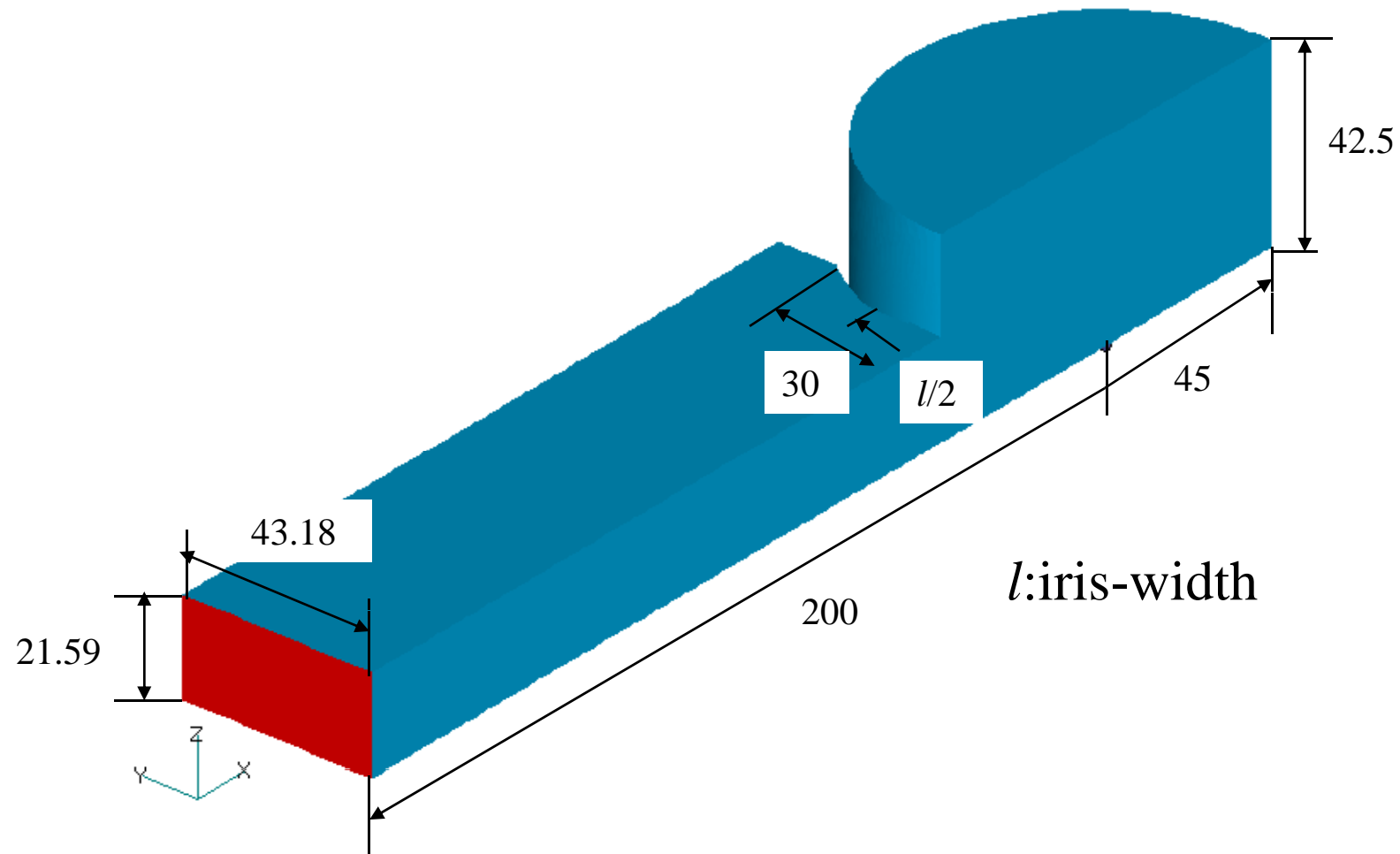
# 電場



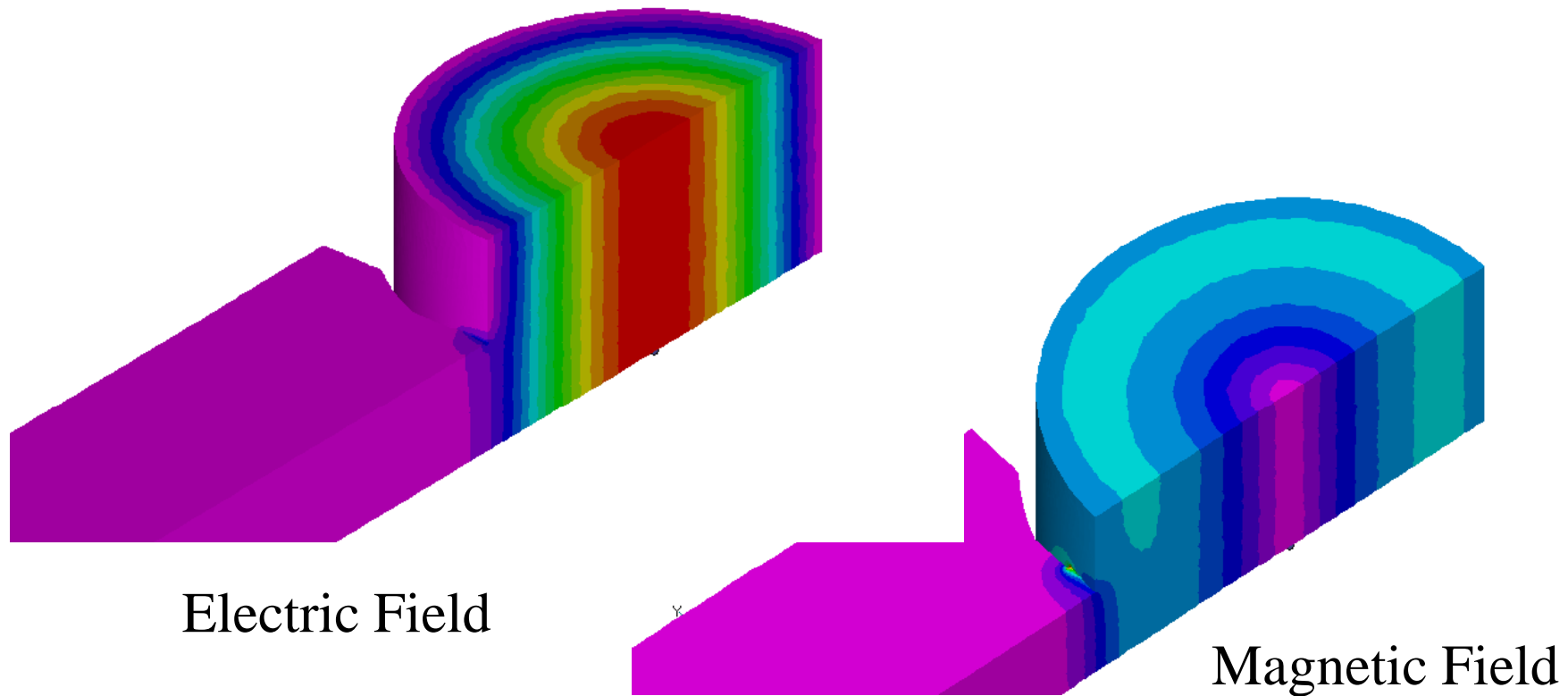
# TEAM Workshop Problem 19 (Microwave Field in a Loaded Cavity)



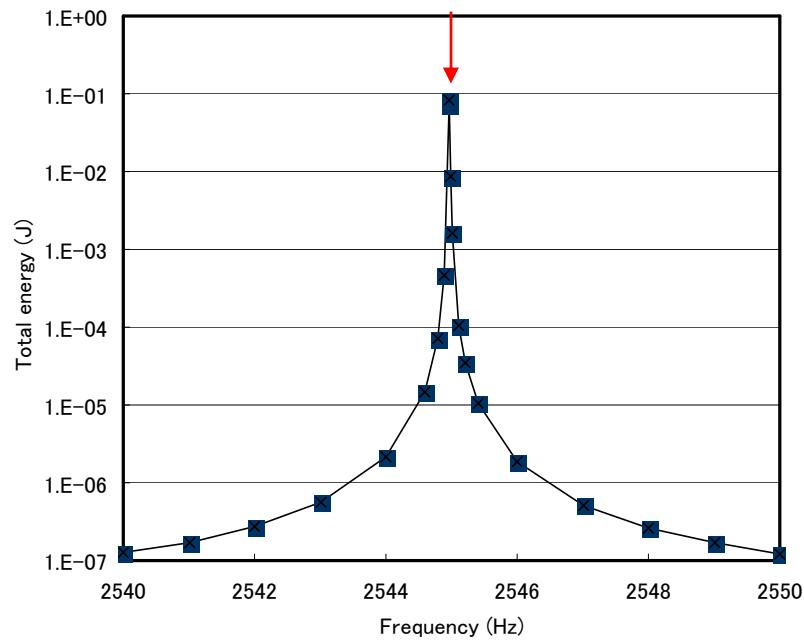
# Problem dimensions (a quarter model)



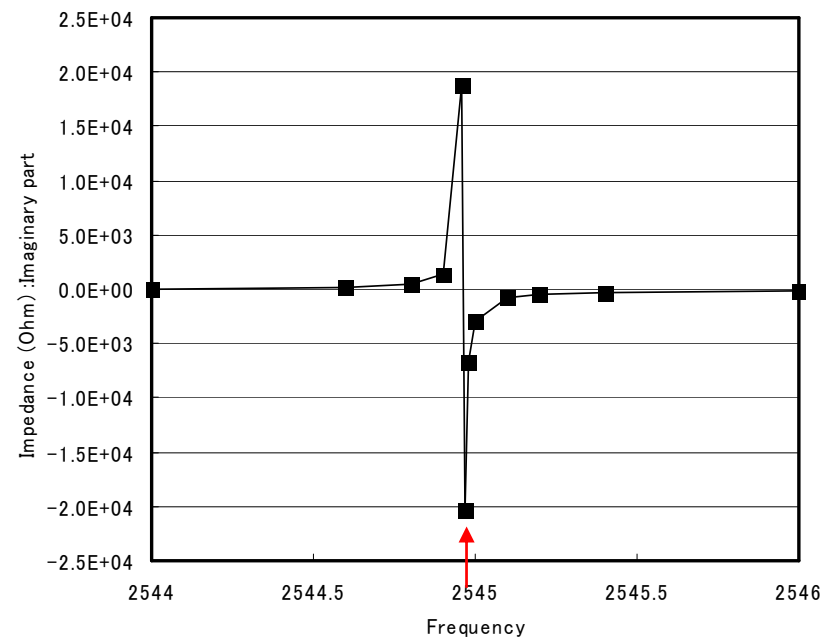
# Fields at resonance (2544.97MHz)



# Total energy and impedance



Total average energy in the cavity and wave guide



Impedance (imaginary part) at the entrance of the wave guide

# Resonant Frequency

iris width  $l=15\text{mm}$ , no load case

- Measured

2545.38MHz (first time)

2644.54MHz (second time)

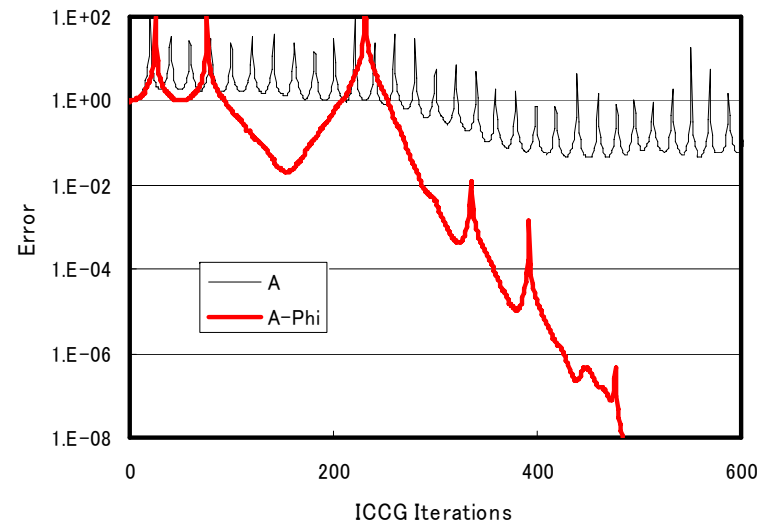
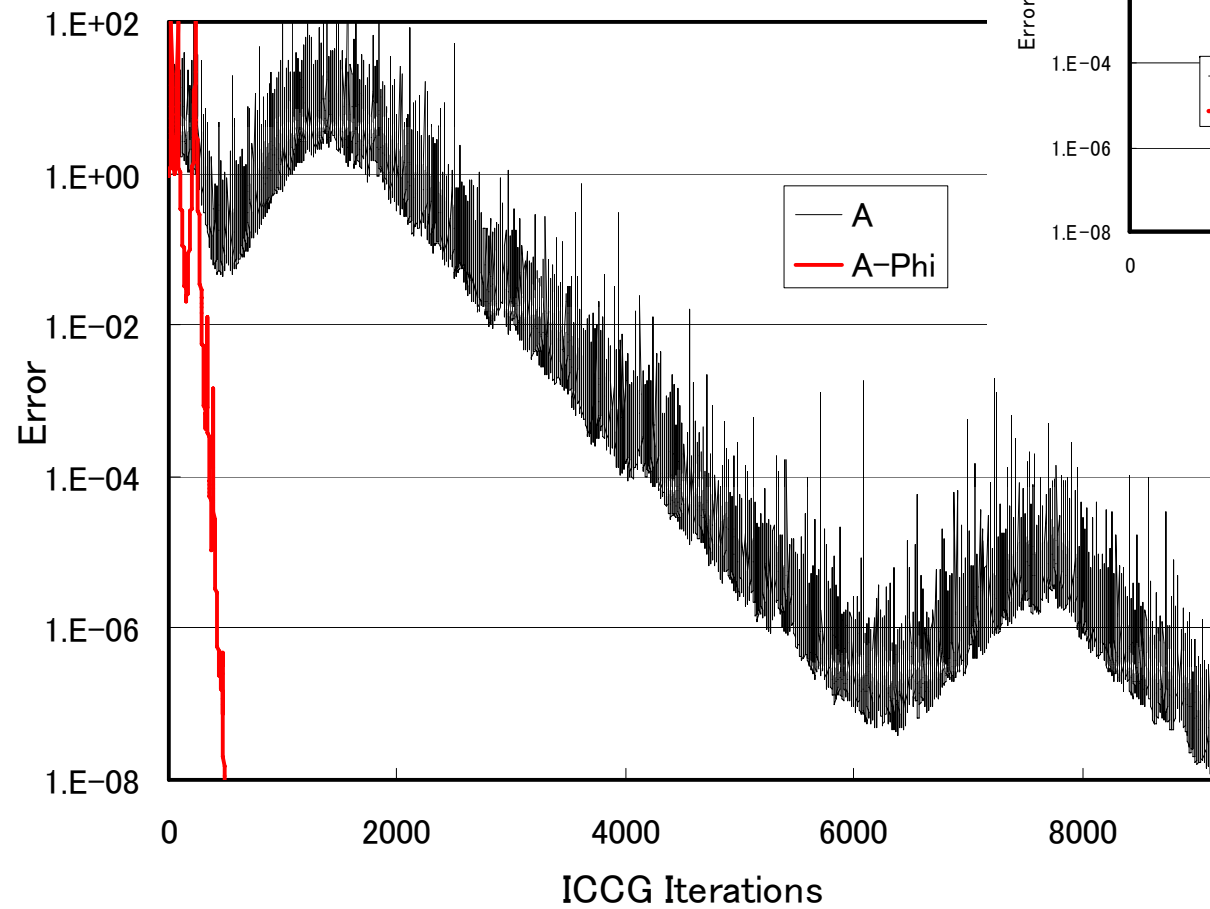
difference =0.84MHz

- Calculated

2544.97MHz

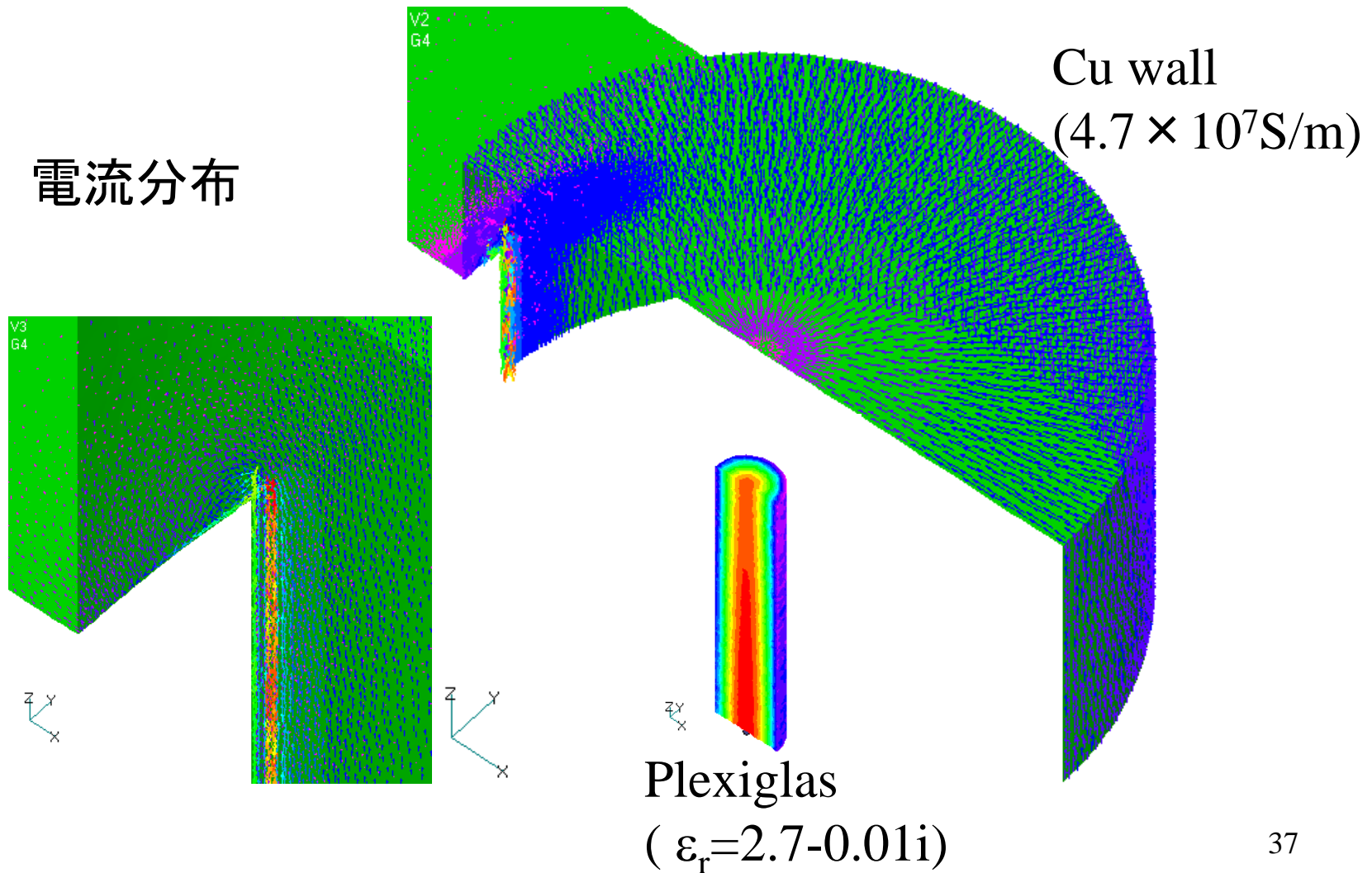
the wall was assumed to be perfectly conducting

# A法とA- $\phi$ 法

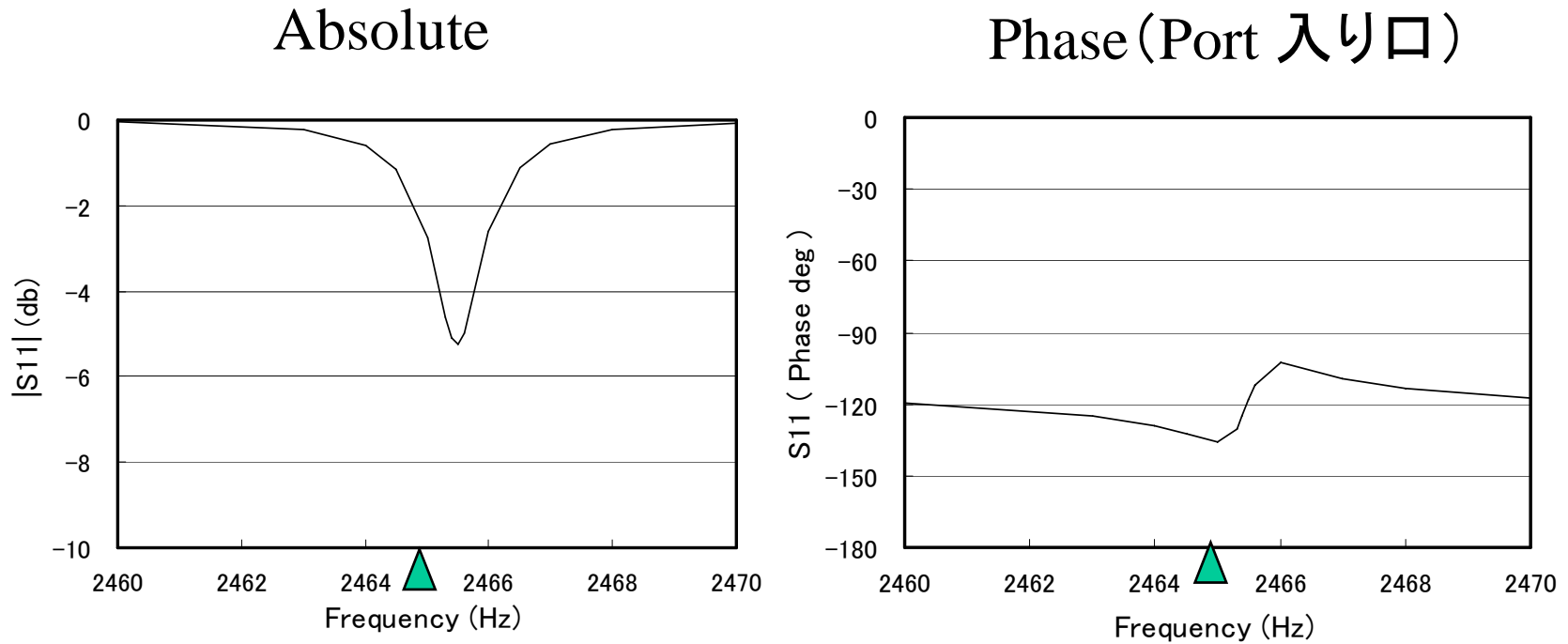


# Loaded case (Plexiglas, $9\phi$ )

電流分布



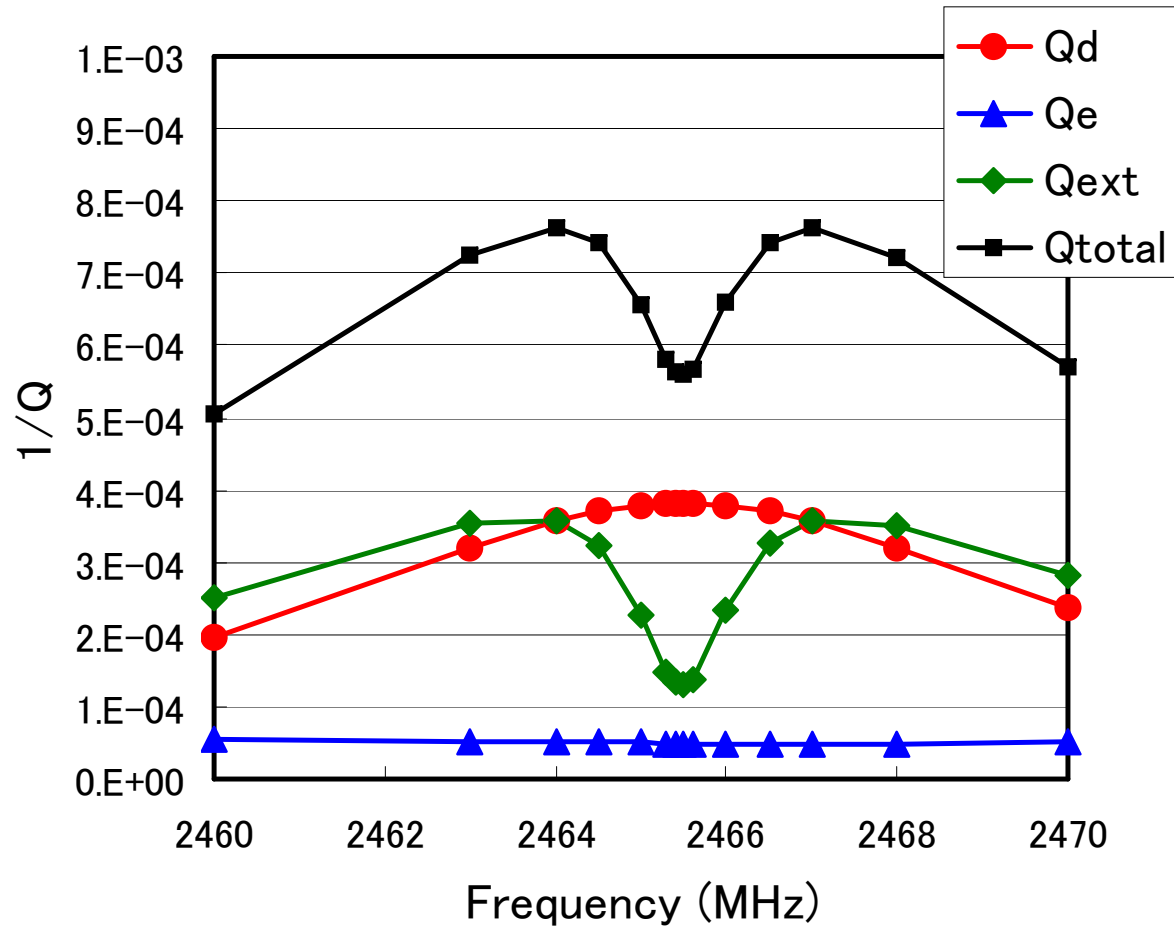
# 反射係数 (S11)



▲ 計測値

There is some uncertainty about these values, and this is the weak point of the benchmark.

# Q値



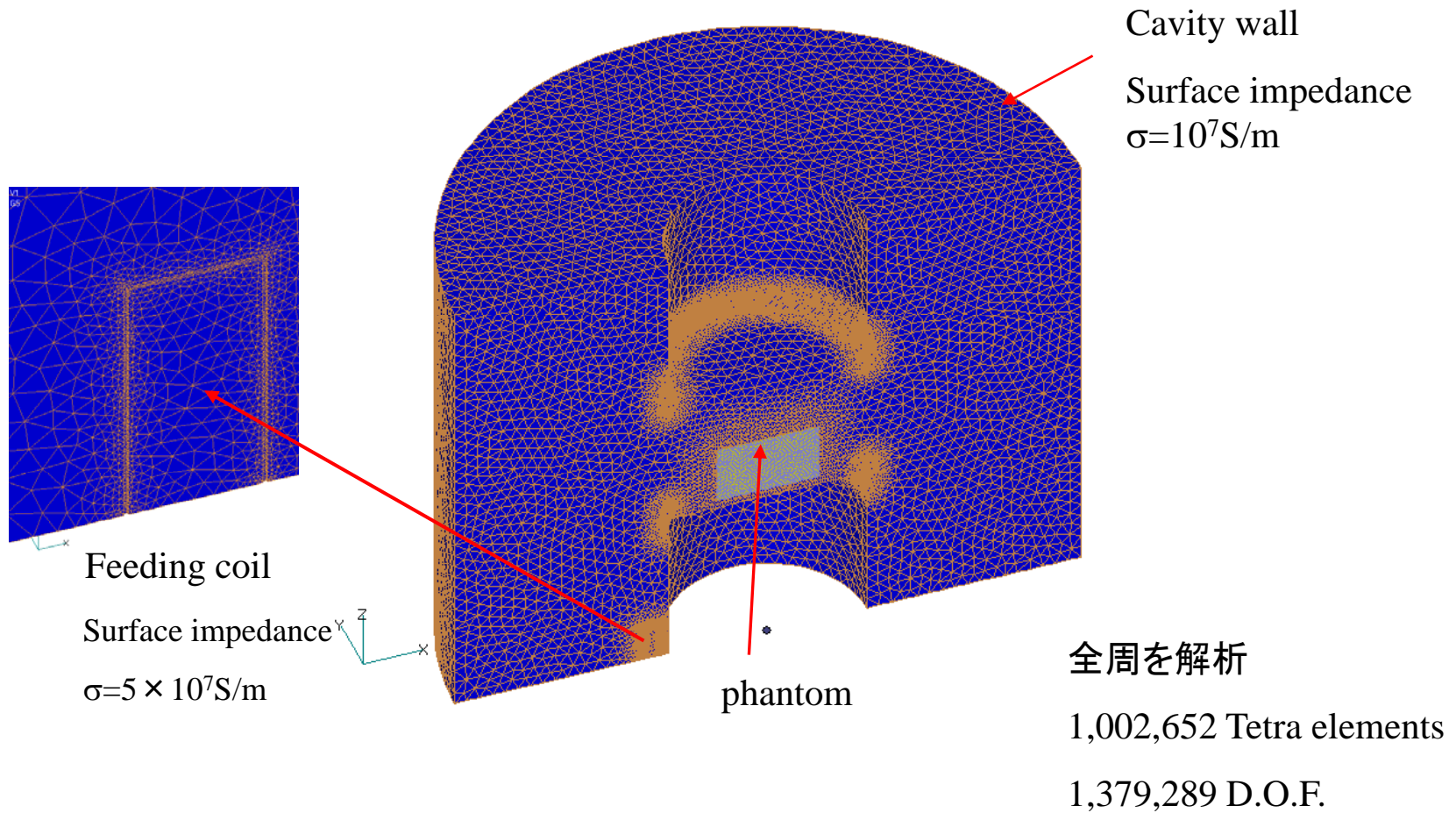
Qd:誘電体損

Qe:導体損

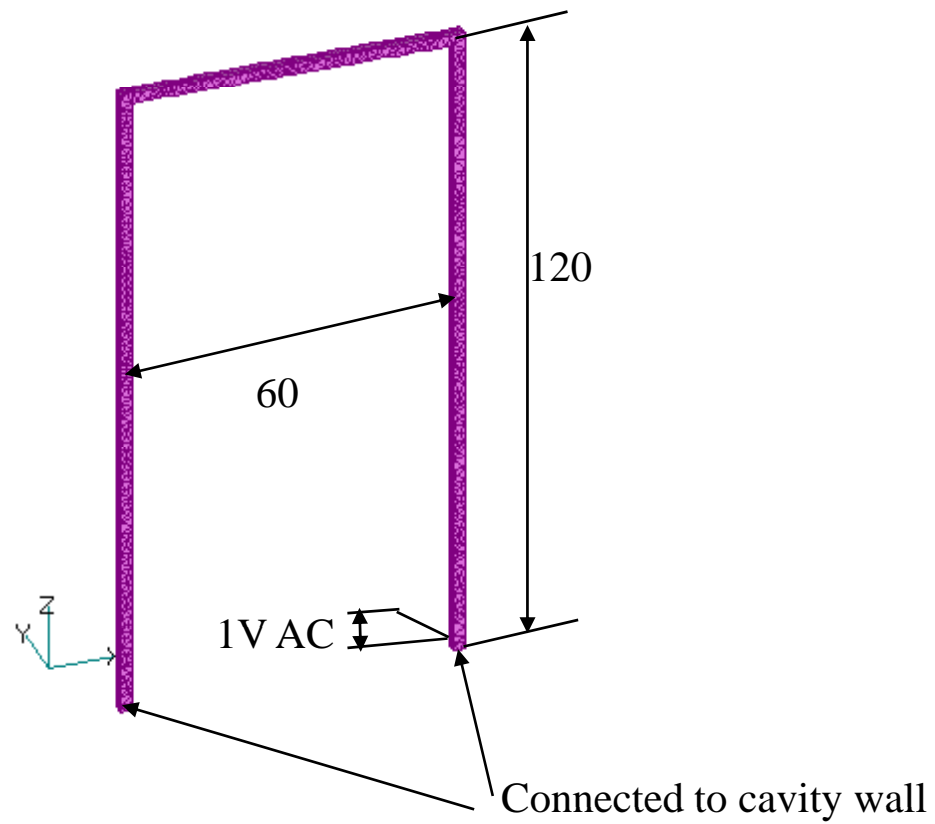
Qext:外部Q

Qtotal:トータル

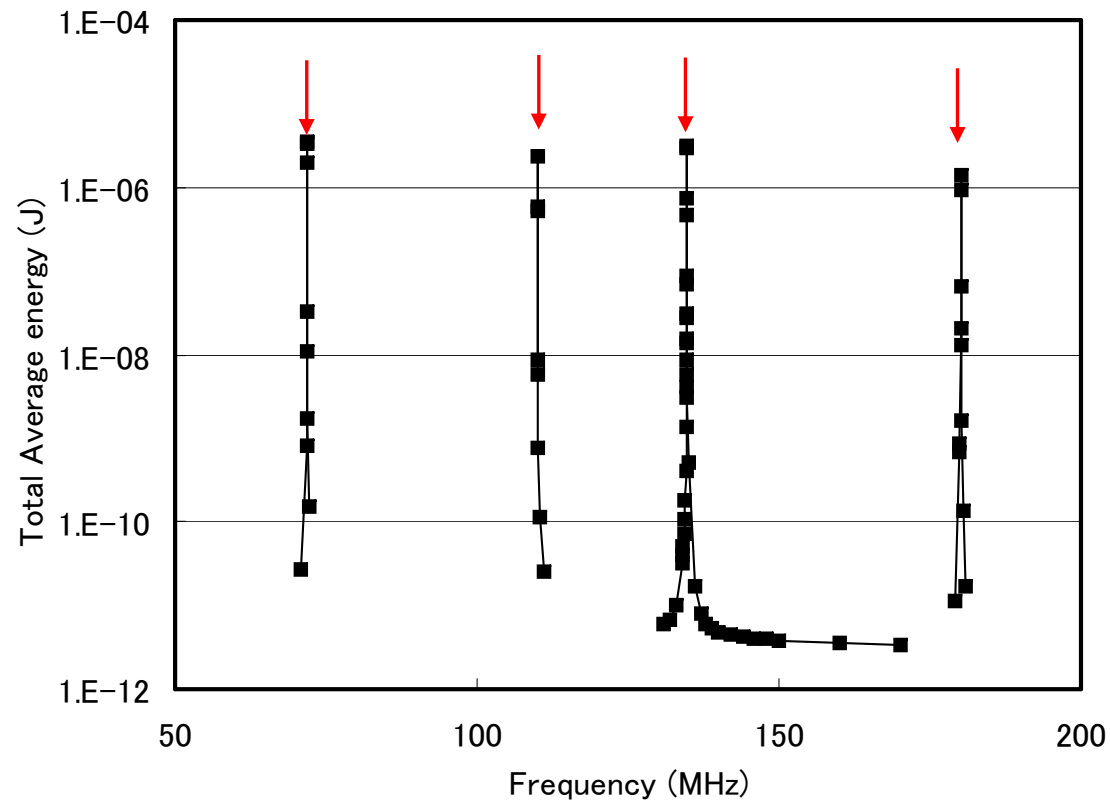
# TEAM Workshop problem 29 (Whole body cavity resonator)



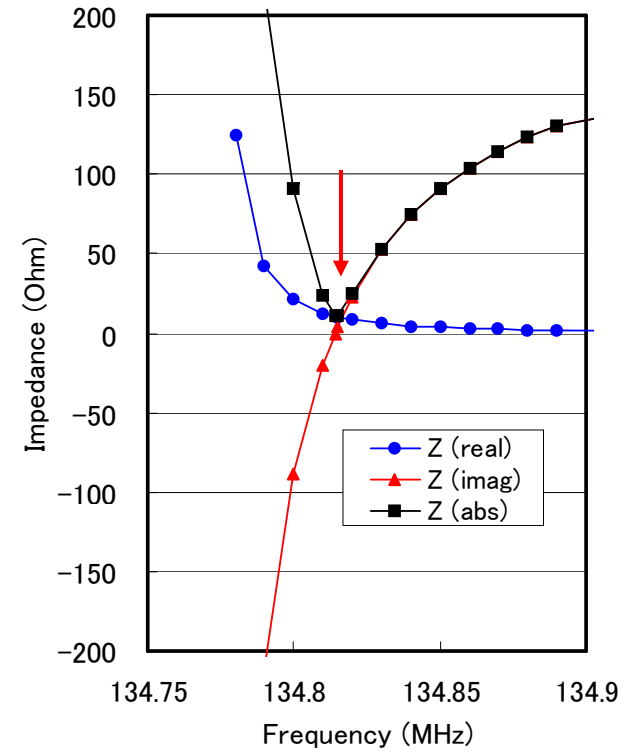
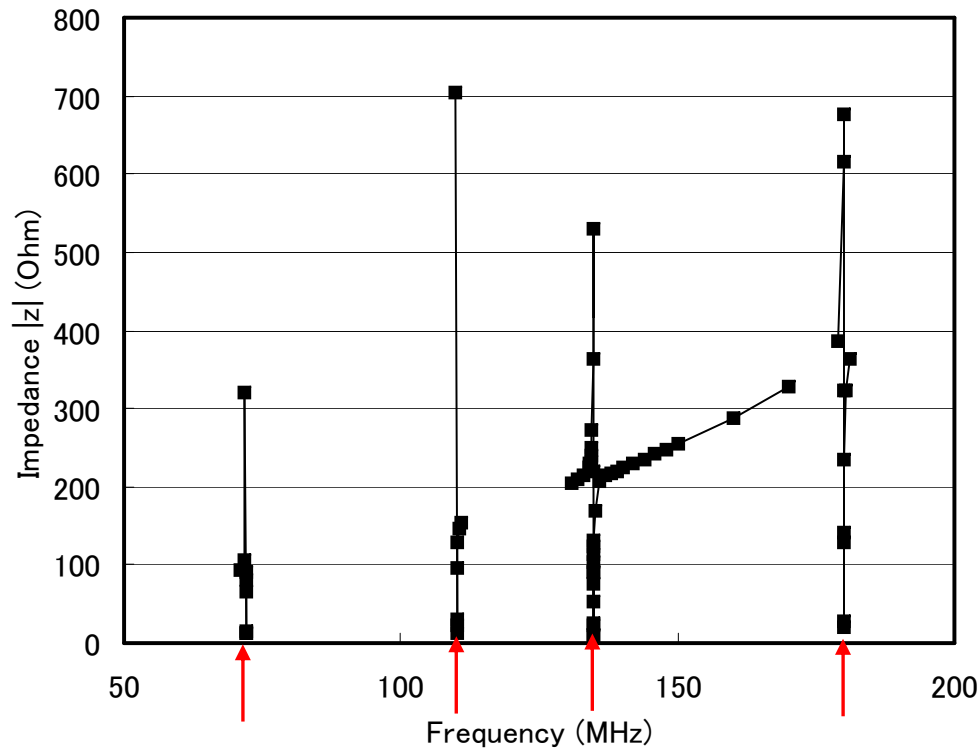
# Feeding coil



# Total average energy (no load case )

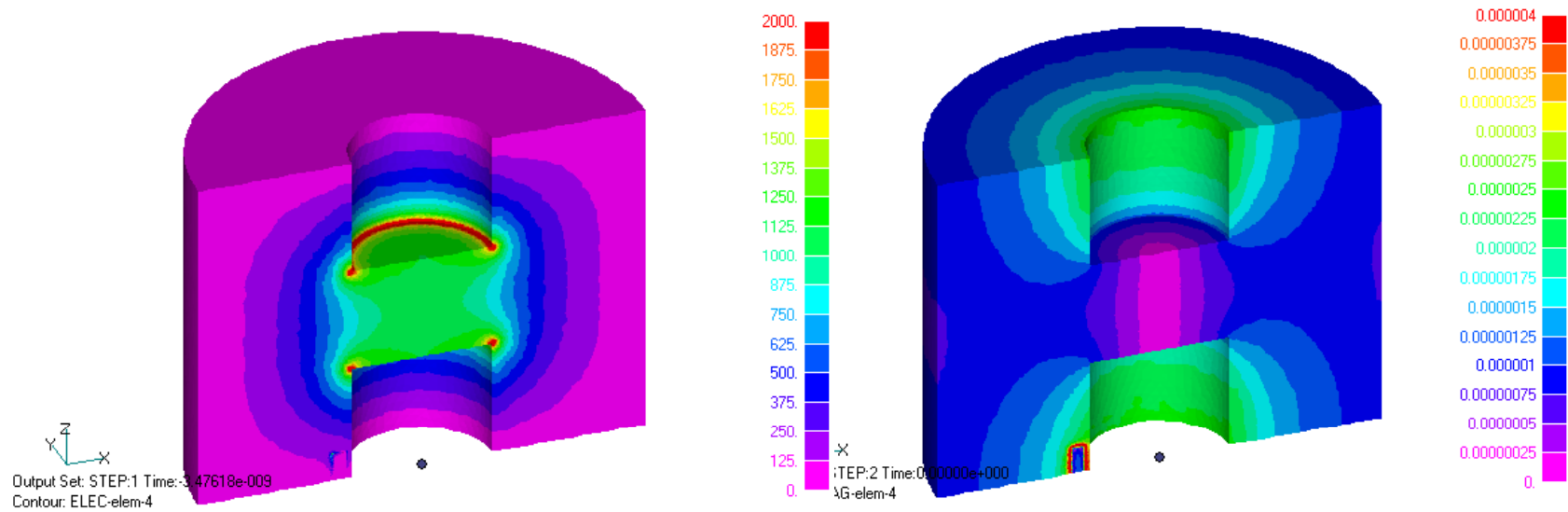


# Impedance (no load case )



Near 3<sup>rd</sup> resonance frequency

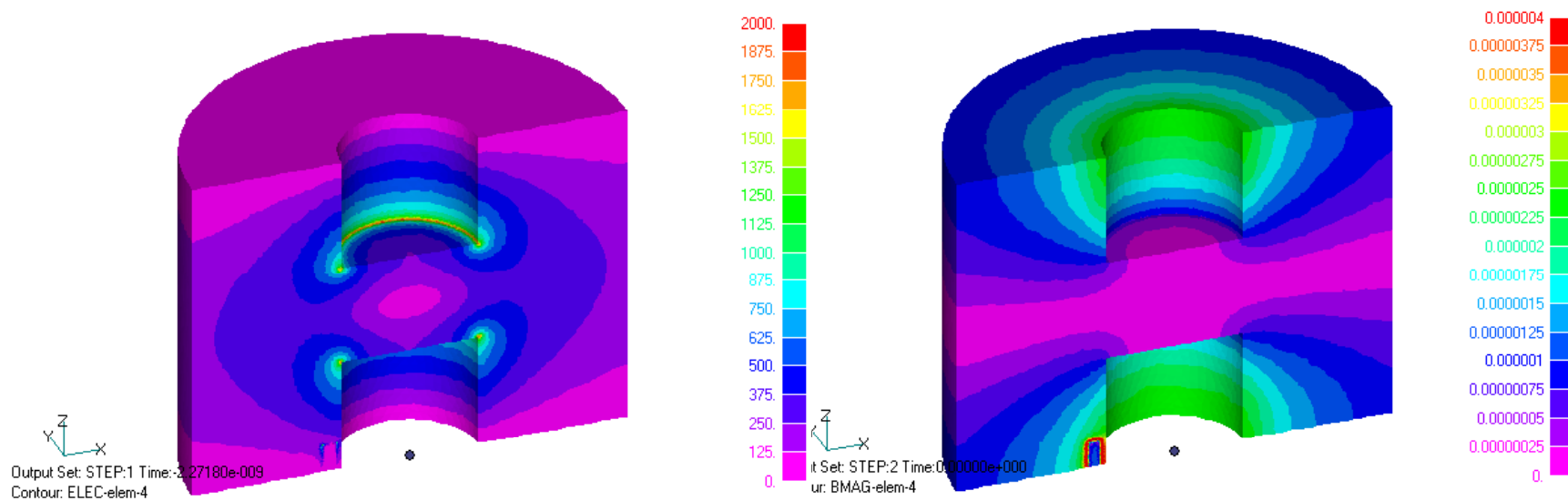
# 1<sup>st</sup> resonant mode (no load case)



電場  $|E|$

磁場  $|B|$

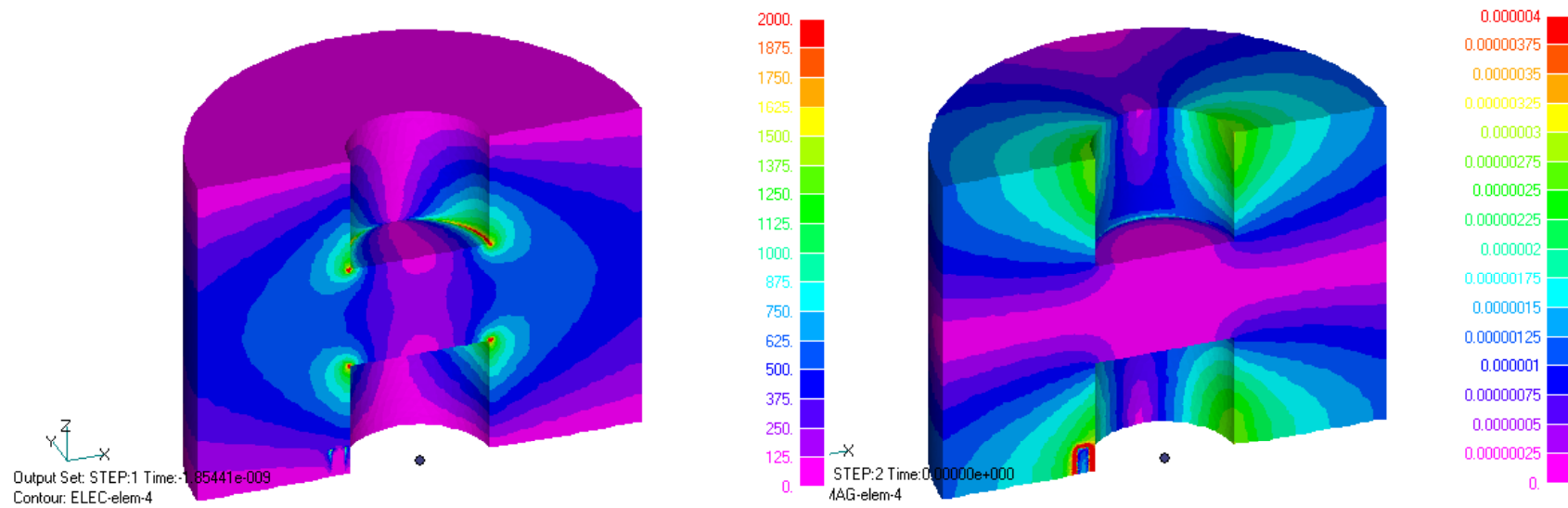
# 2<sup>nd</sup> resonant mode (no load case)



電場 |E|

磁場 |B|

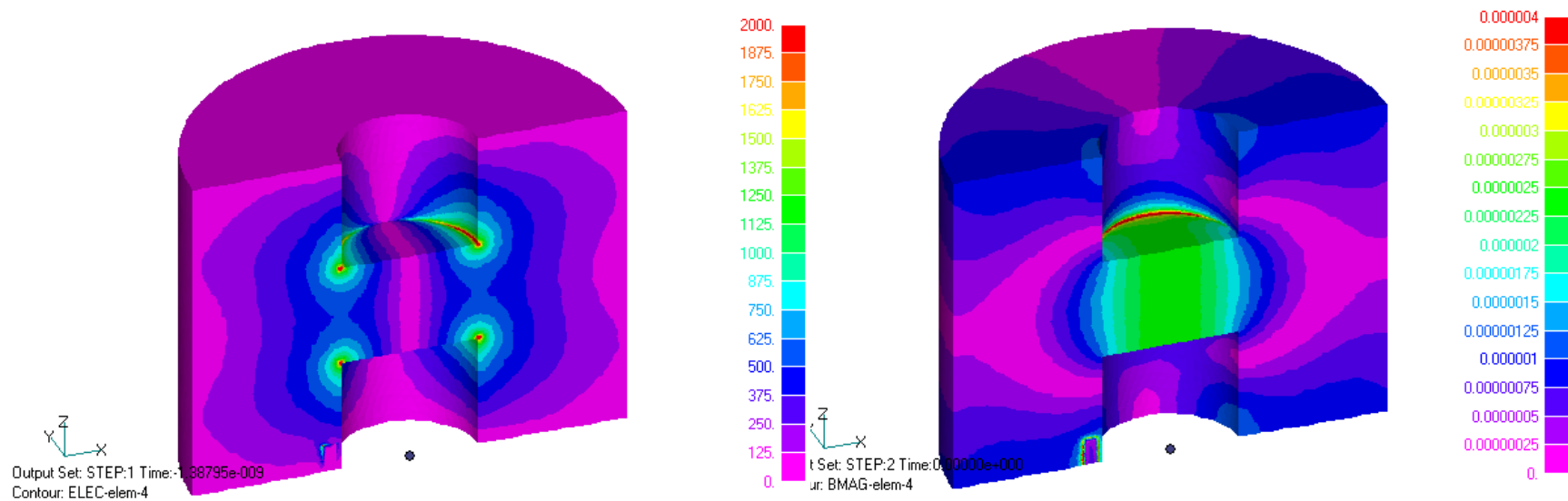
# 3<sup>rd</sup> resonant mode (no load case)



電場  $|E|$

磁場  $|B|$

# 4<sup>th</sup> resonant mode (no load case)



電場  $|E|$

磁場  $|B|$

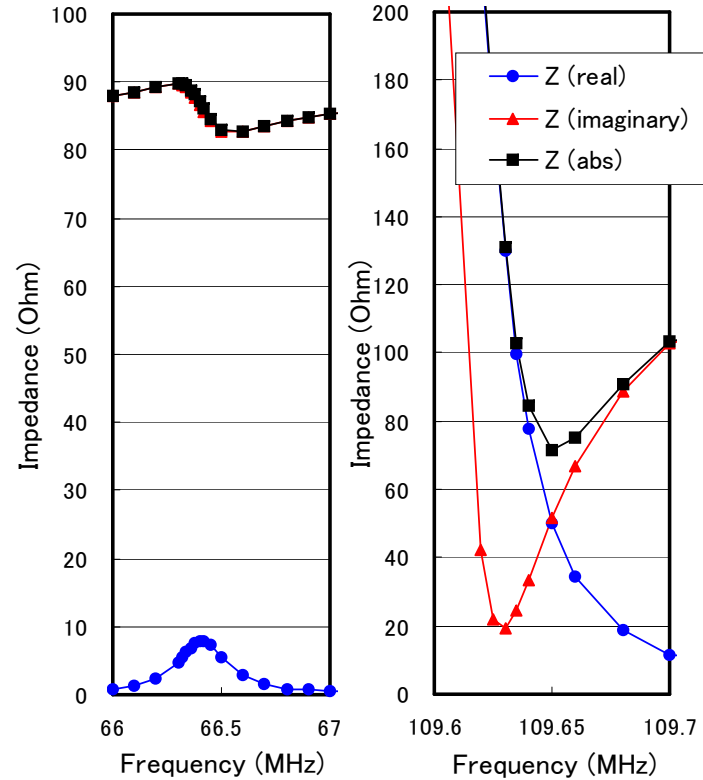
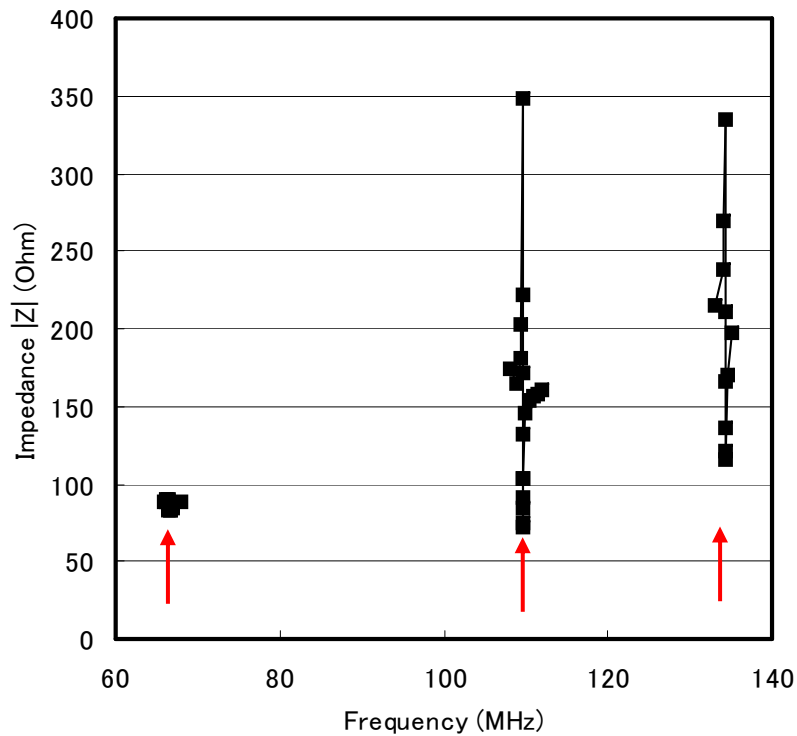
## Resonant frequencies (no load case)

Resonance mode	Measured	Calculated
1st	77	71.918
2nd	113	110.045
3rd	134	134.814
4th	N/A	180.122

In MHz

# Impedance ( with phantom )

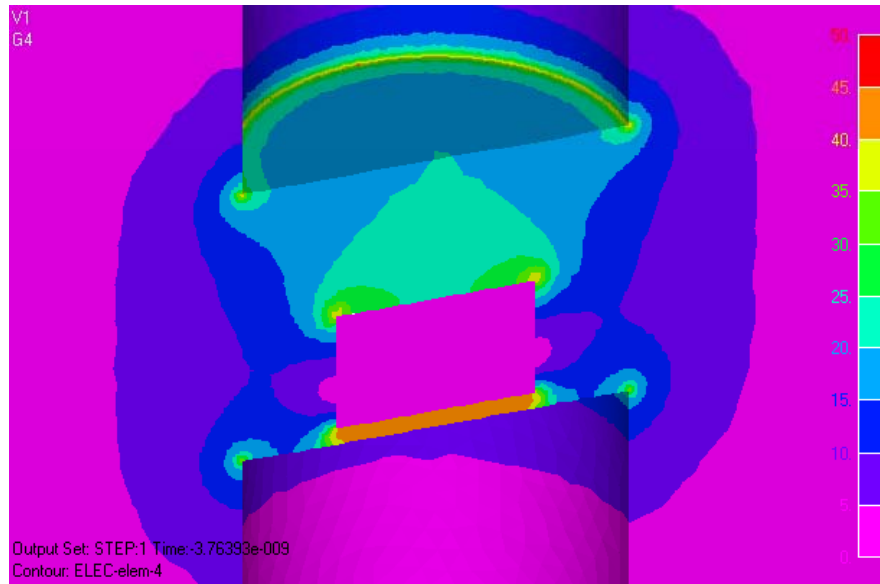
$$\epsilon_r=80, \sigma=0.52\text{S/m}$$



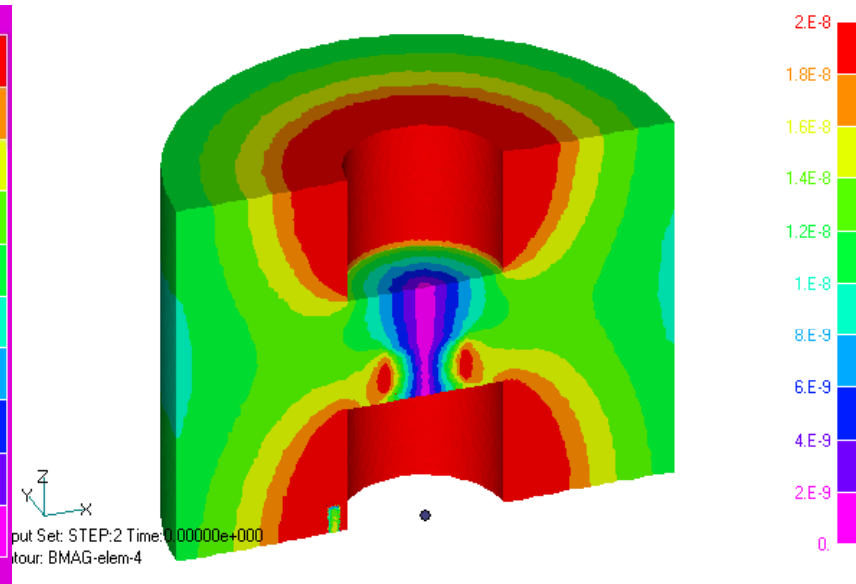
Near 1<sup>st</sup>  
resonance  
frequency

Near 3<sup>rd</sup>  
resonance  
frequency

# 1<sup>st</sup> resonant mode (with phantom)

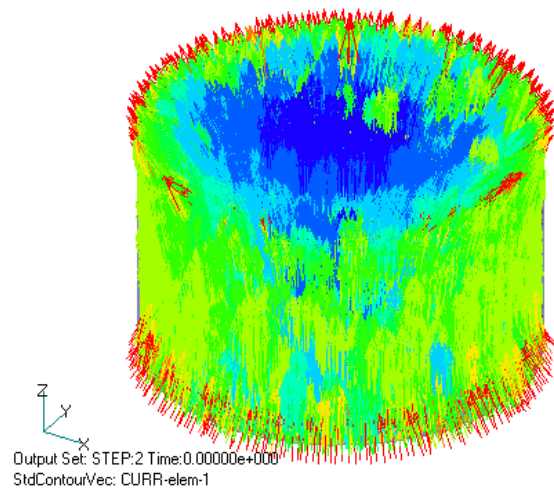


電場  $|E|$

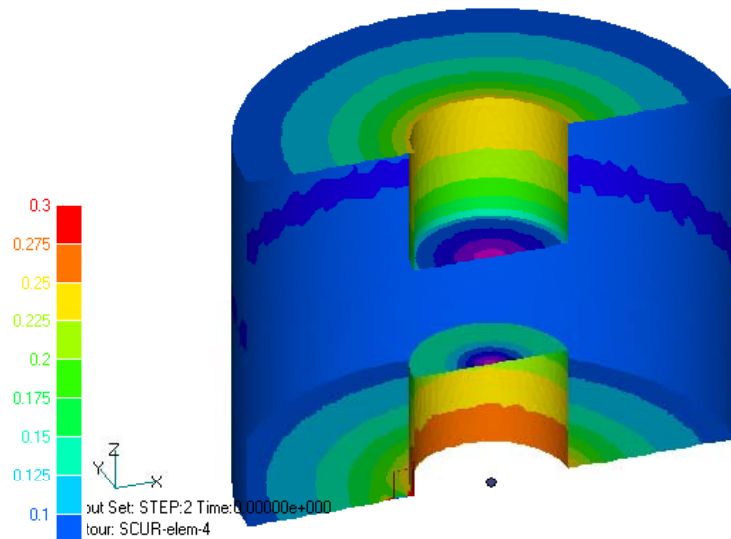


磁場  $|B|$

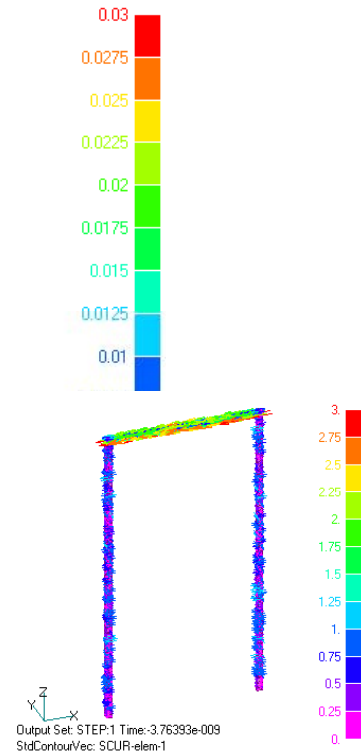
# Current distribution ( at 1<sup>st</sup> resonant frequency with phantom )



phantom



Cavity wall



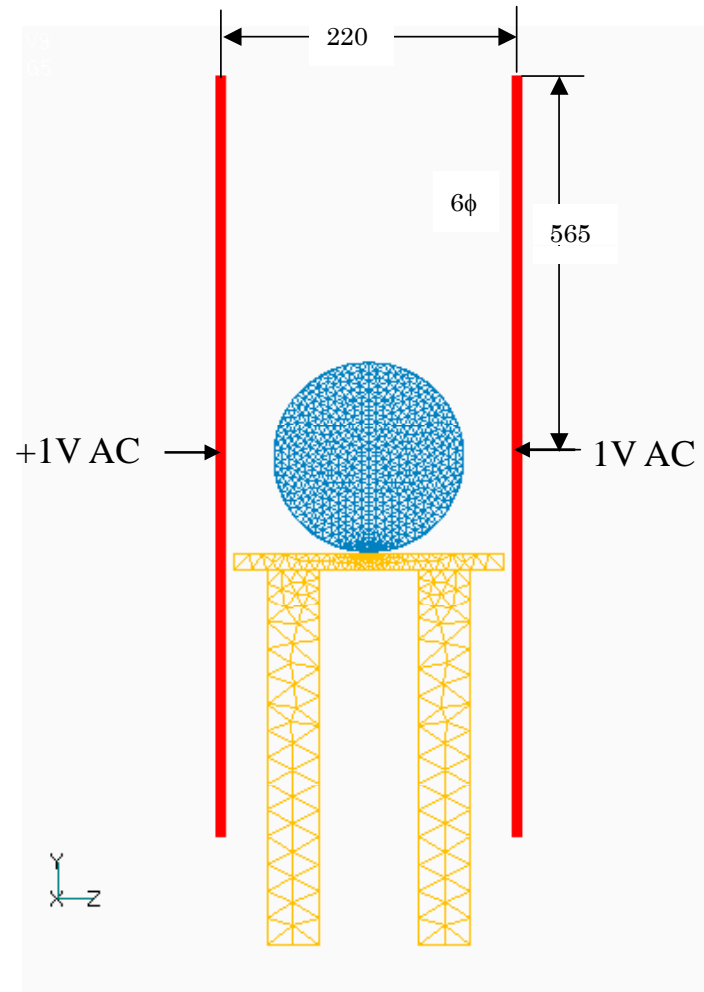
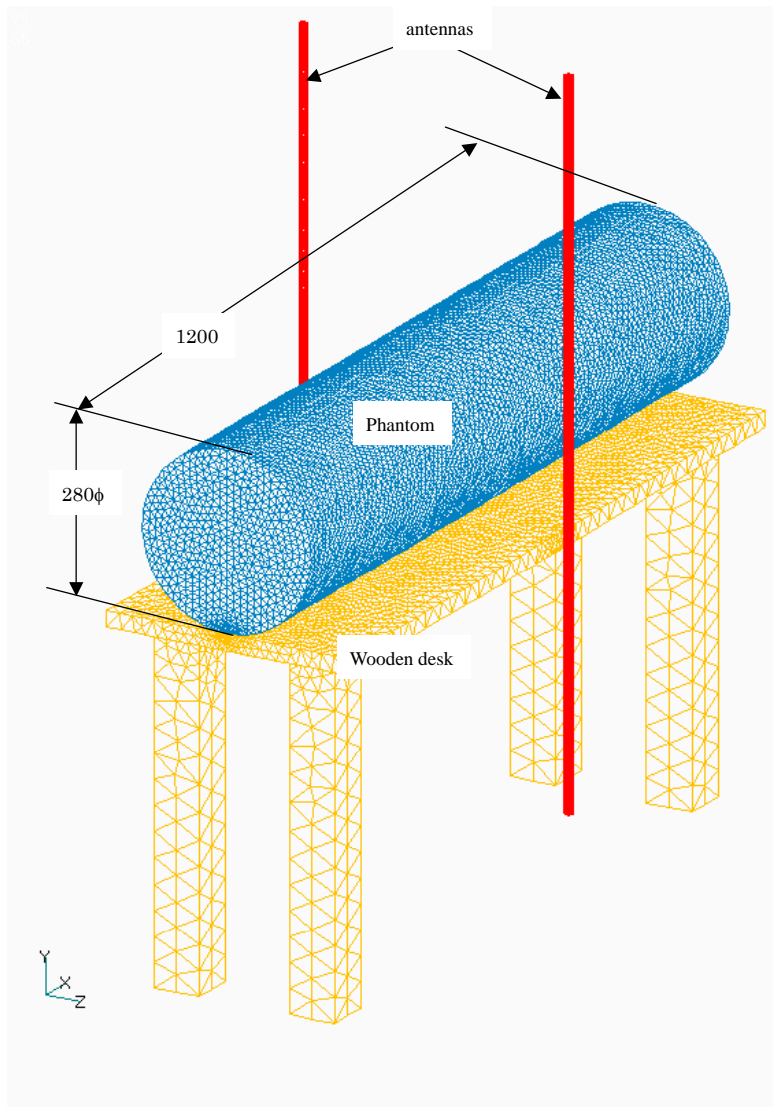
Feeding coil

# Resonant frequencies (with phantom)

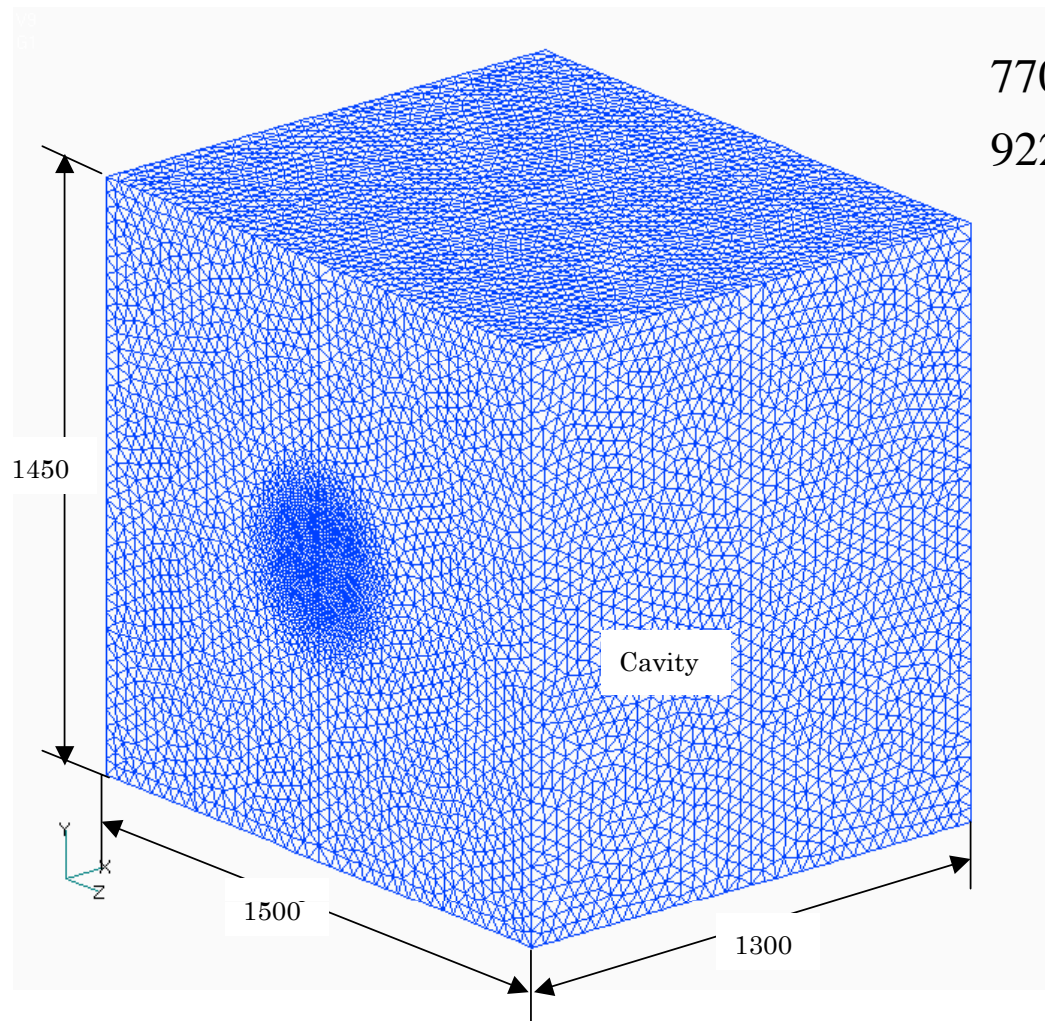
Resonance mode	Measured	Calculated
1st	68.6	66.42
2nd	110	109.63
3rd	134	134.41

in MHz

# Resonant cavity applicator with multiple antennas for a hyperthermic treatment

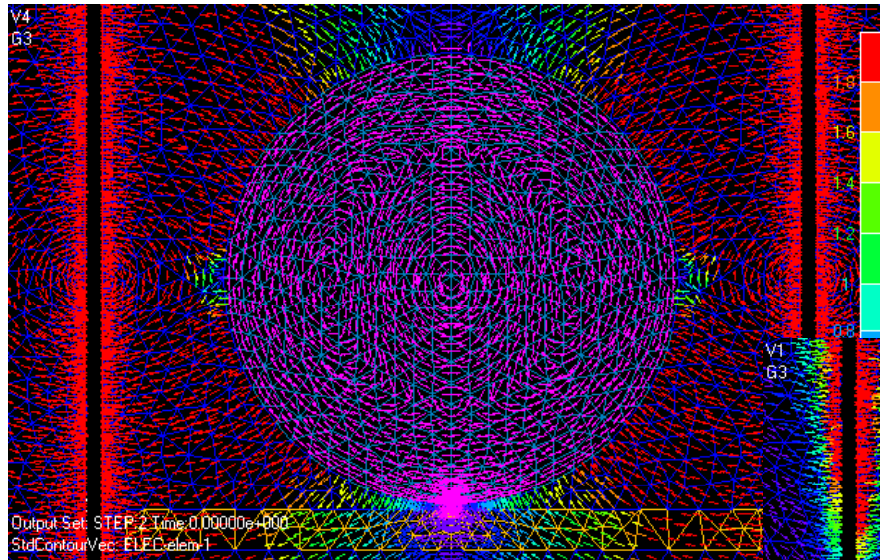


# Model mesh ( cavity wall )



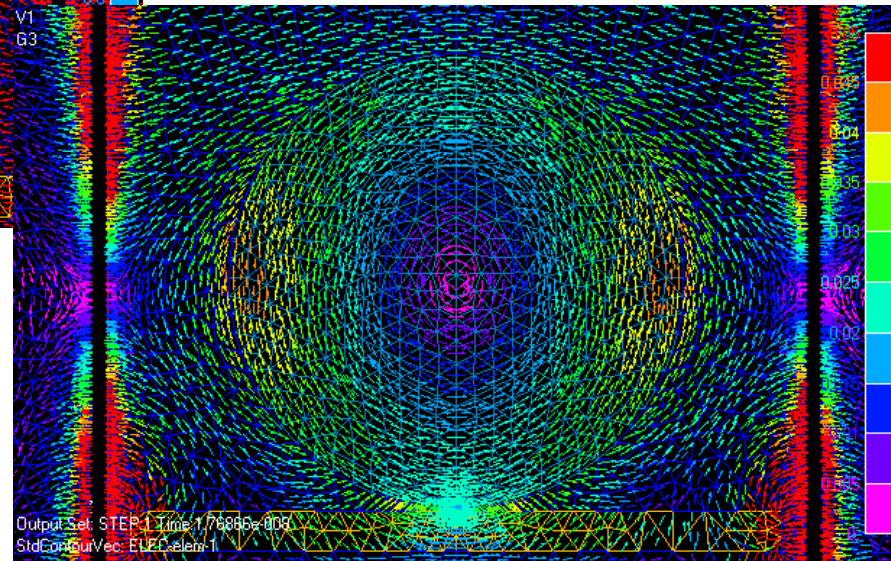
770028 tetra elements  
922386 D.O.F.

# Electric field (opposite phase )

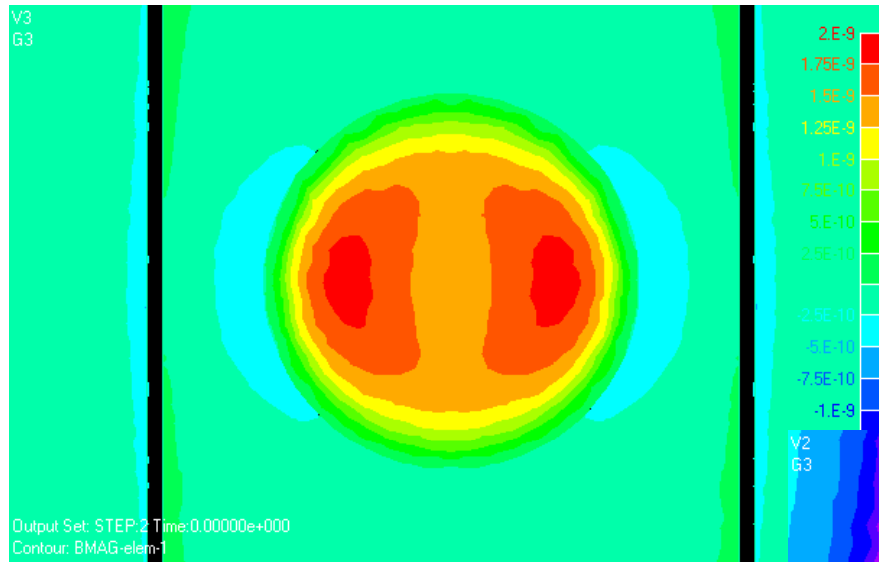


Real part

Imaginary part

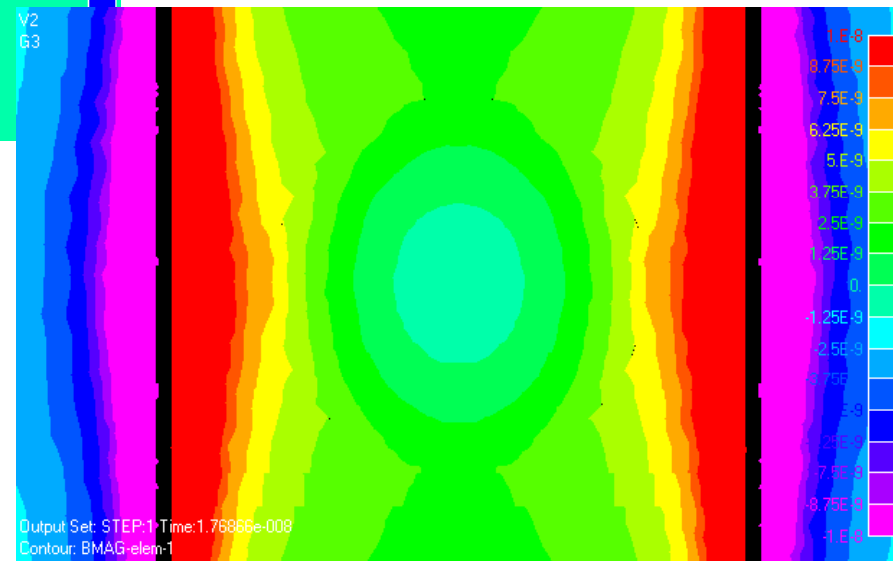


# Magnetic field ( opposite phase )

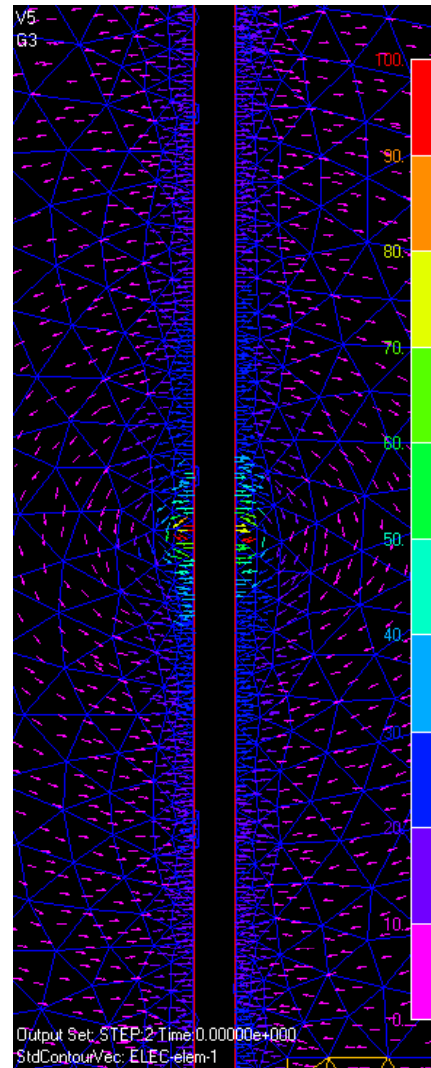


Real part

Imaginary part

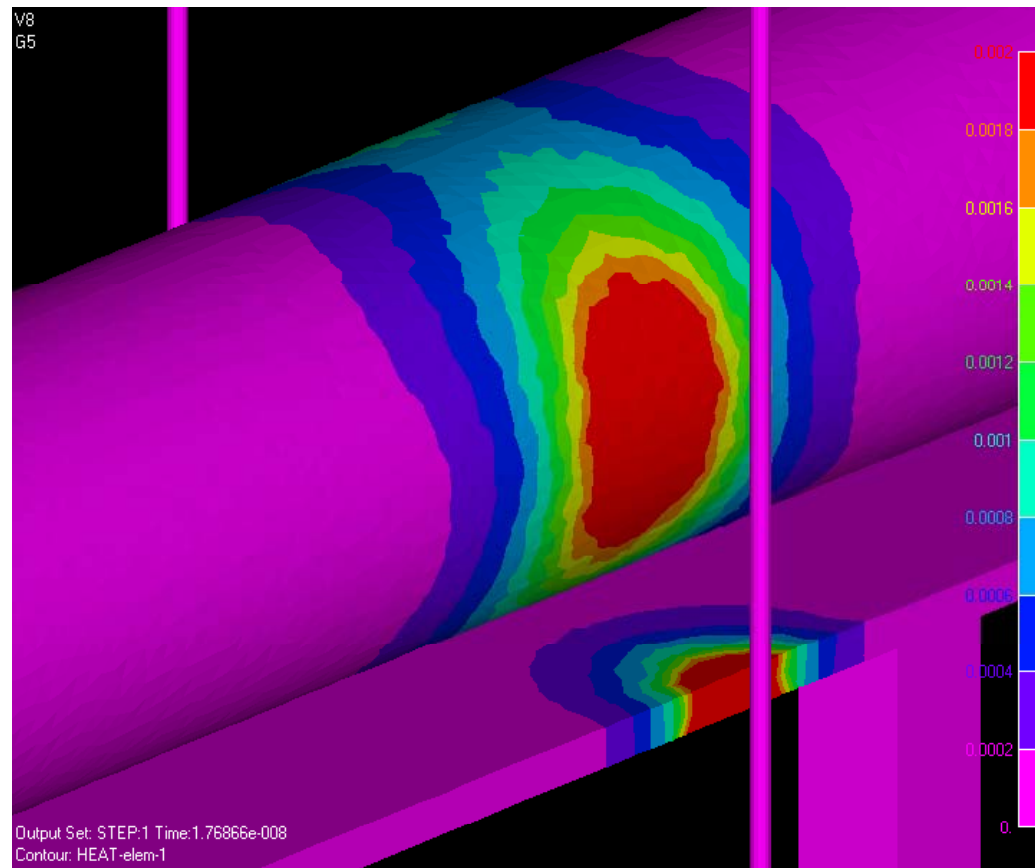


# Electric field near antenna

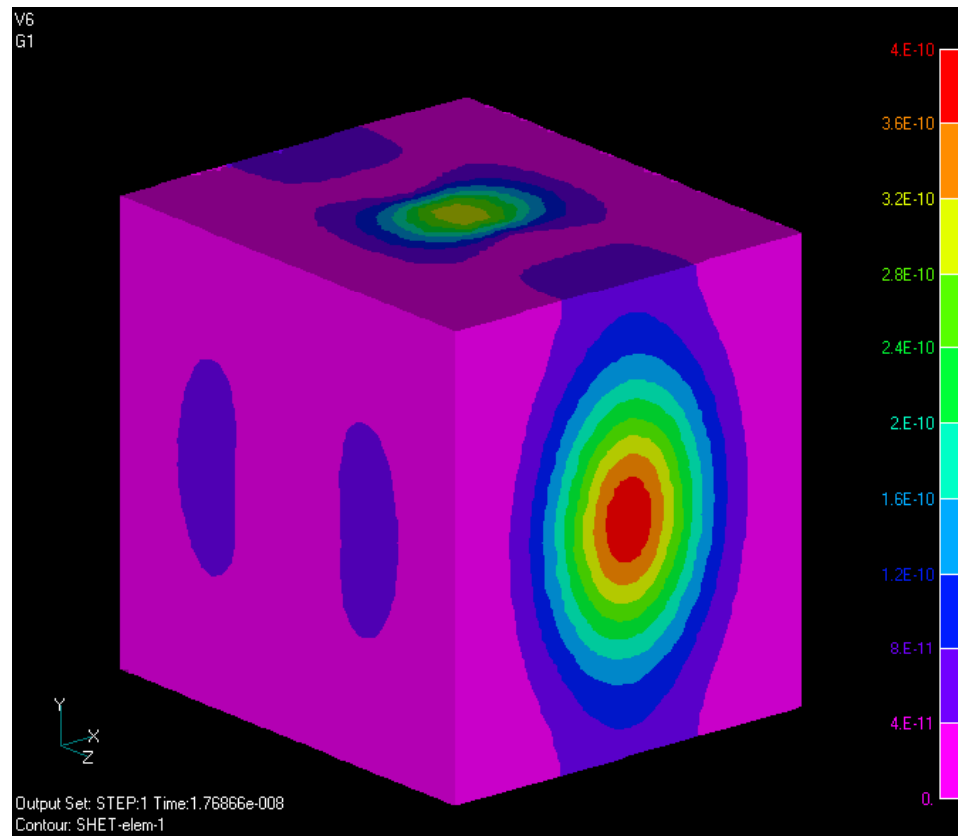


Real part

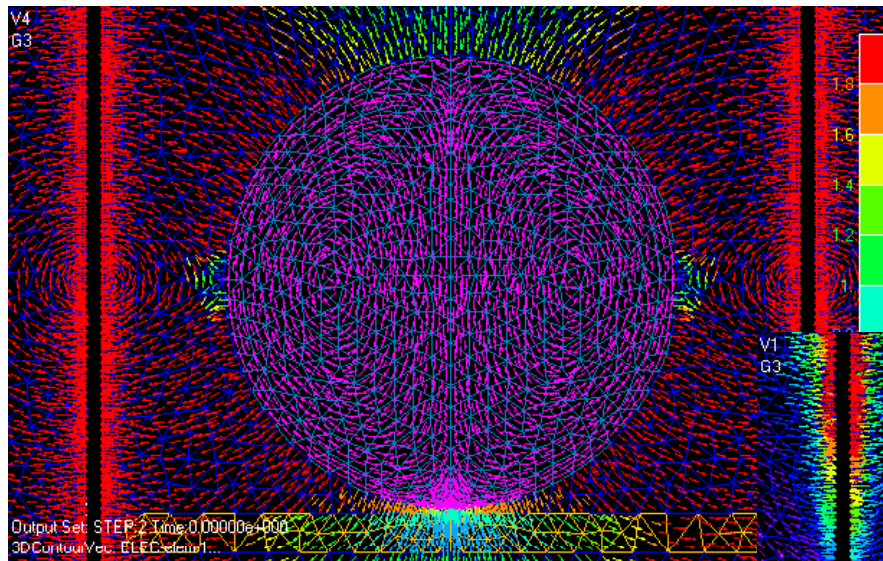
# Joule heating in phantom and table ( opposite phase)



# Joule heating in cavity wall ( opposite phase )

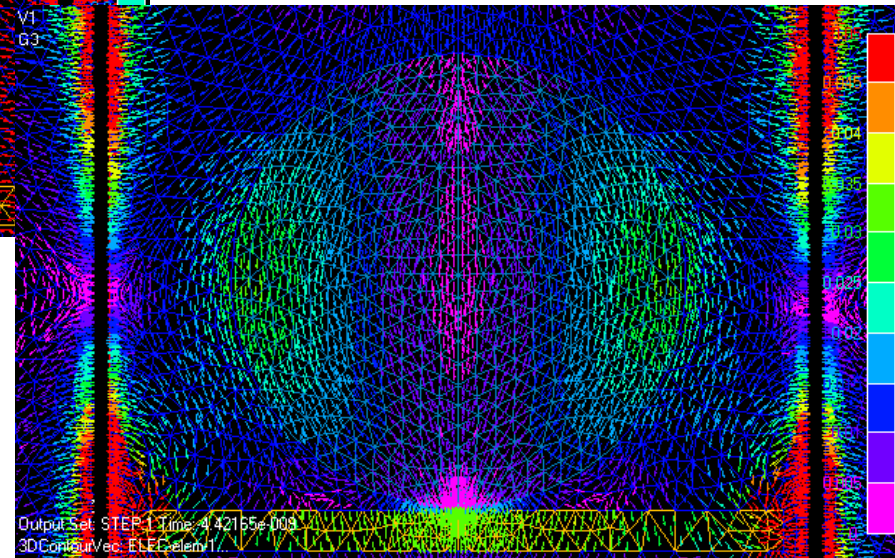


# Electric field (same phase )

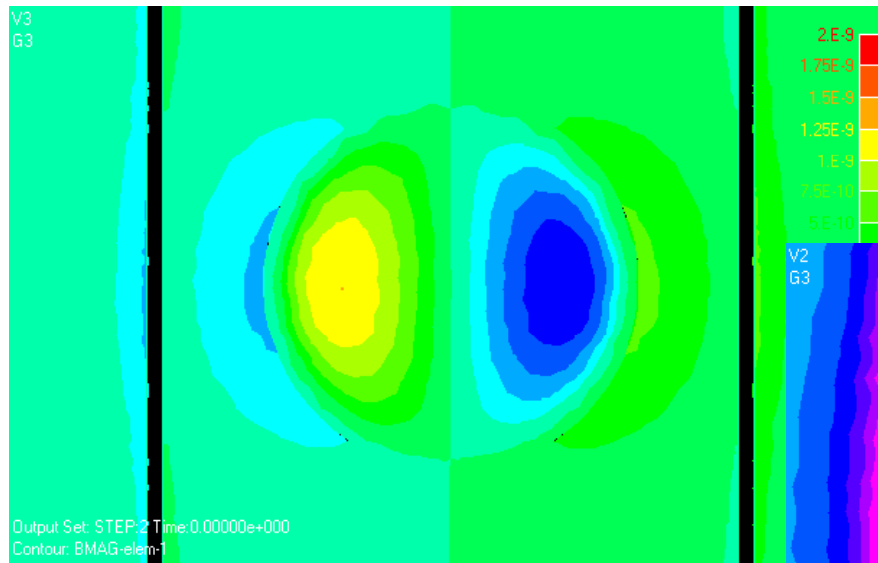


Real part

Imaginary part

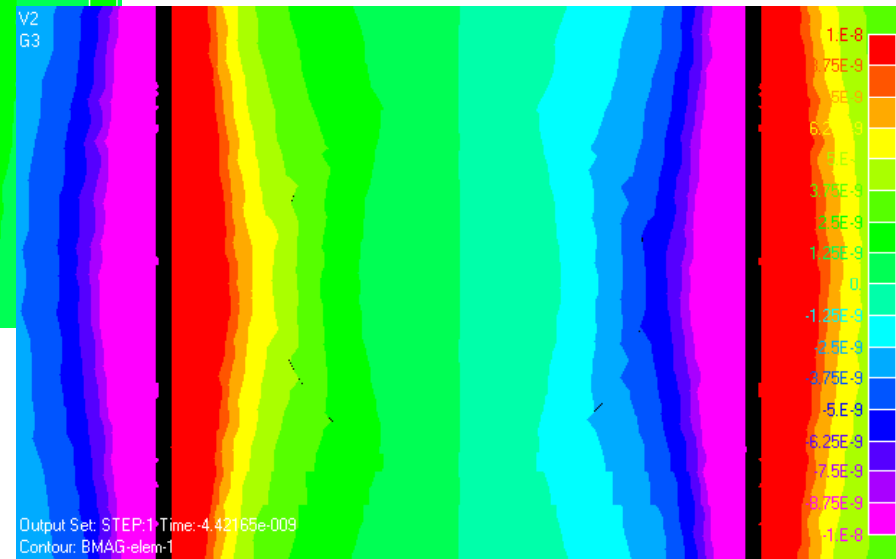


# Magnetic field ( same phase )

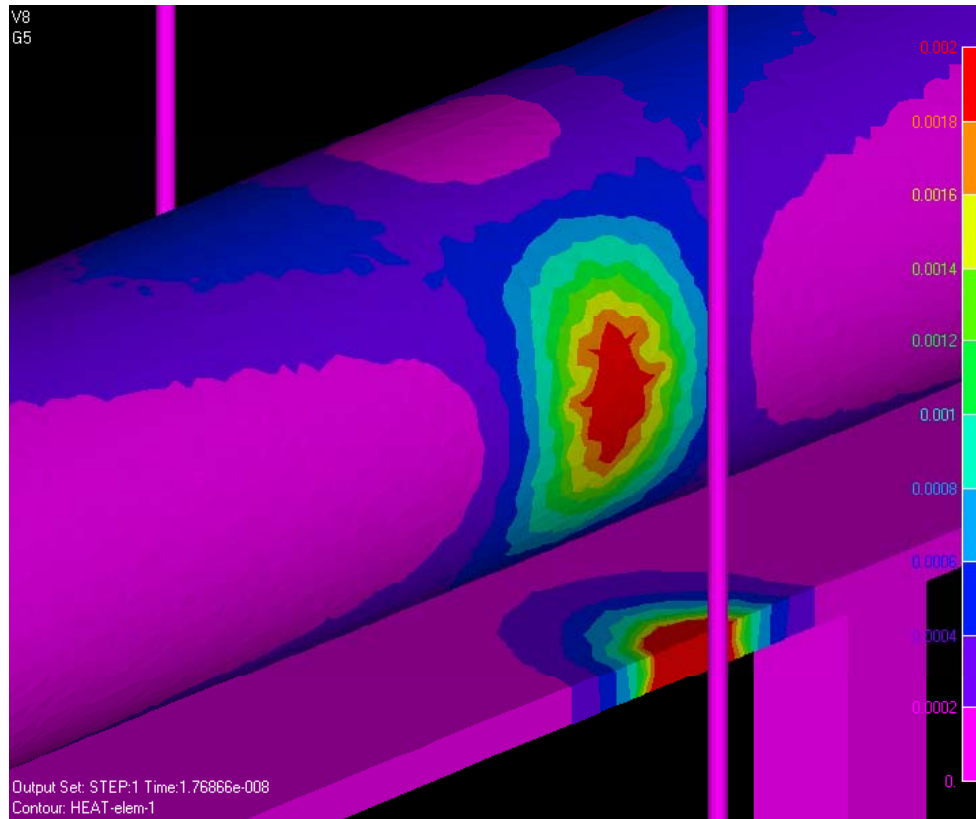


Real part

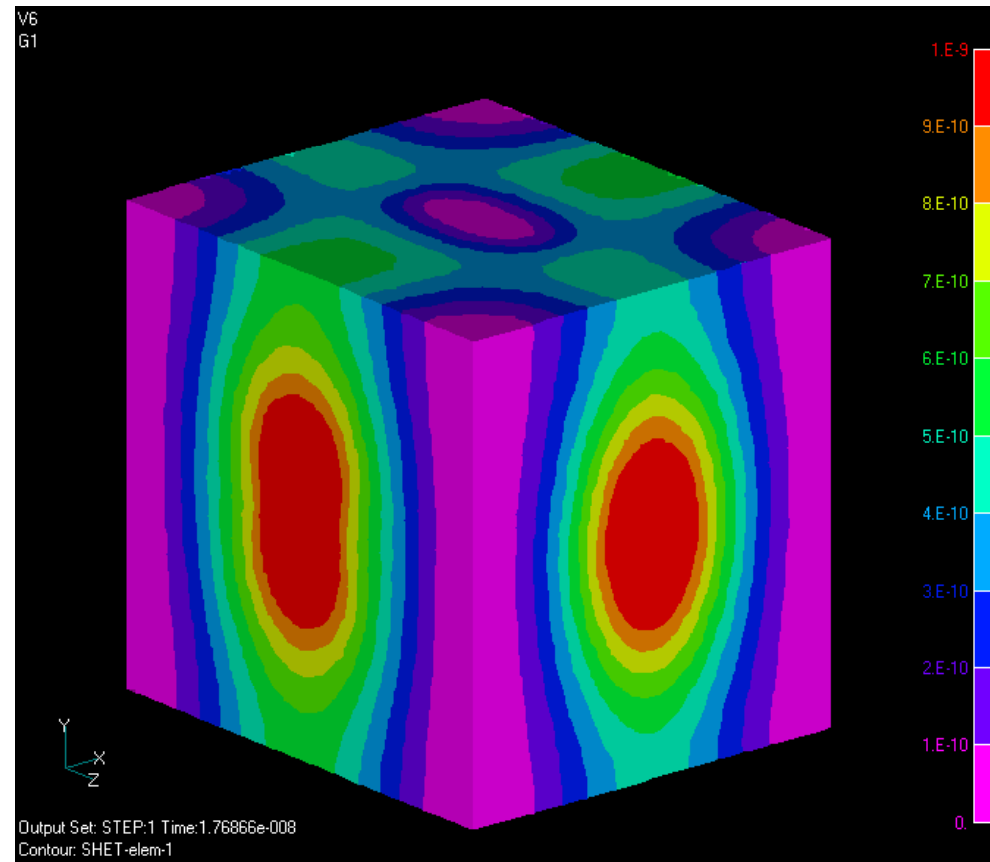
Imaginary part



# Joule heating in phantom and table ( same phase)



# Joule heating in cavity wall ( same phase )



# Analysis summary

	Same phase	Opposite phase
Number of ICCG Iterations	422	485
Calculation time (s)	1470.8	1672.3
Antenna input current (sum of two antenna,)		
Real part (A)	2.090E-05	2.171E-05
Imaginary part (A)	4.512E-03	4.540E-03
Total average magnetic energy (J)	1.650E-12	1.337E-12
Total average electric energy (J)	8.001E-12	7.727E-12
Total Joule loss(W)		
Phantom	9.366E-06	9.816E-06
Table	9.742E-07	9.341E-07
Cavity	4.322E-09	5.842E-10
Antenna	1.050E-07	1.060E-07
Total	1.045E-05	1.086E-05

4, 2.58GHz, 2Gbyte RAM

1 V AC is assumed to be applied in antenna gap.

# 今後の課題

- 開領域問題
  - PMLの導入, 外部照射, 遠方解
- メッシュジェネレータの高度化
  - Adventureプログラムの利用
- Adaptive GMG