# Magnetic Field Analysis Using Hybrid Mesh of Linear Hexahedral and Tetrahedral Edge Elements

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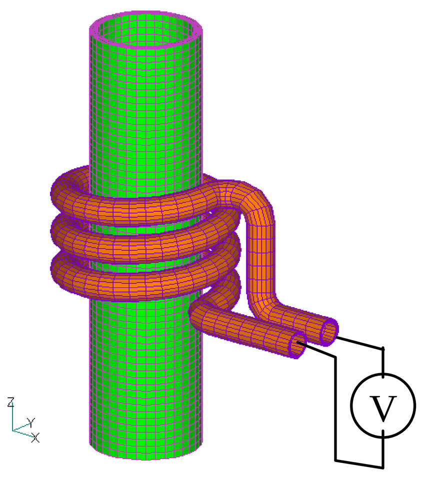
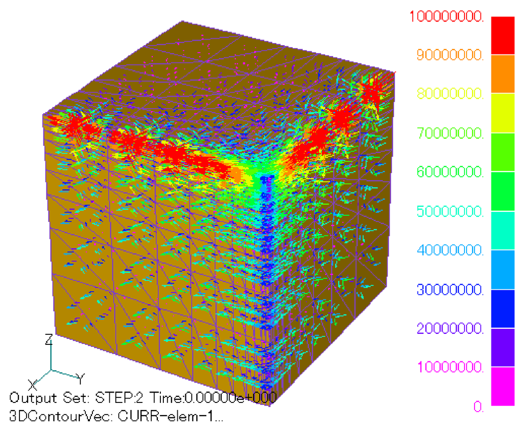
## How do we mesh and solve it?

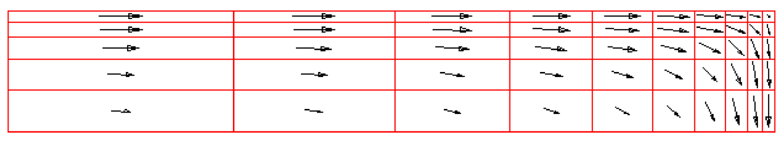



•Skin depth of eddy current λ < Thickness of structure
Surface of the conductor must be divided into thin layers.
→ Hexahedral mesh
•Geometry of air region is complicated.
Delaunay automatic meshing is desirable.
→Tetrahedral mesh
→ **Hybrid mesh**

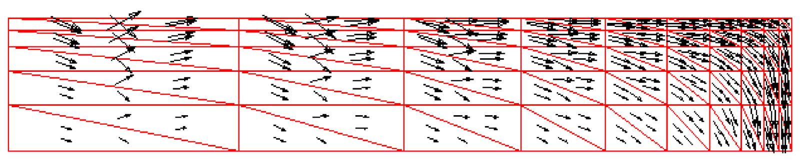
●Using thin linear tetrahedral elements in conductors
→ Inaccurate eddy current and heat loss



Linaer Hexahedral



Linear Tetrahedral



### How are hexahedral and tetrahedral elements joined?

●Inserting pyramidal elements

Not easy to handle. Most pre-post processors do not treat pyramidal elements.

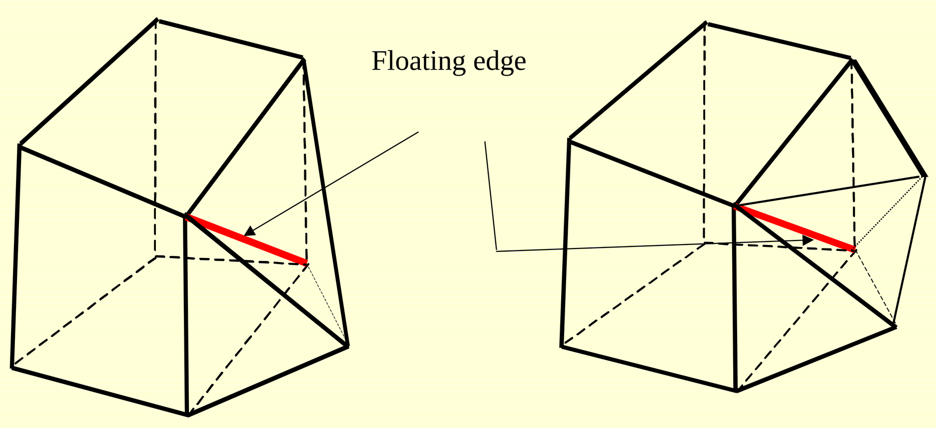
●Non-conformal constraints of linear tetra and hexa

Easy to to be implemented and effective.

●Conformal constraints of higher order elements

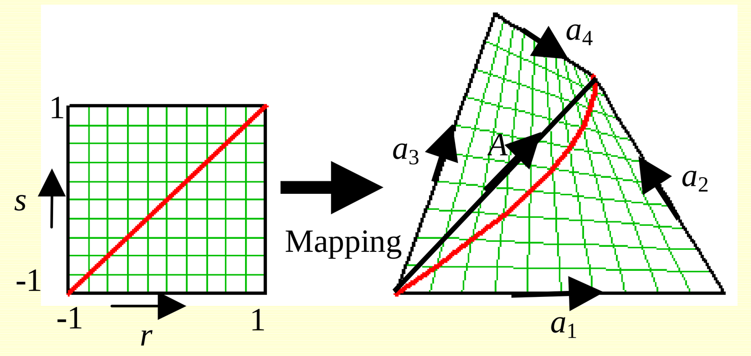
More difficult to be implemented.

### Method of Joining Hexahedral and Tetrahedral Elements




■Only floating edges between hexahedral and tetrahedral elements and no floating nodes in mesh

### Non-conformal constraint




Equate magnetic fluxes through tow triangles.

$$-A+a_1+a_2=A-a_3-a_4$$

↓

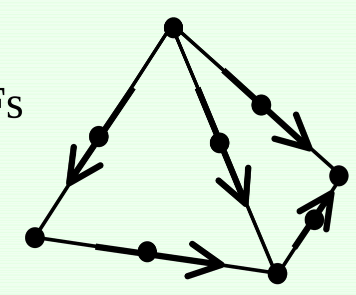
$$A=(a_1+a_2+a_3+a_4)/2$$

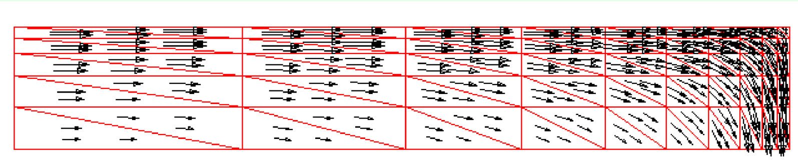
●Normal components of magnetic flux density are continuous when the quadrilateral is parallelepiped, but not continuous otherwise
●Total flux is conserved.
●Tangential components of magnetic vector potential and electric field are discontinuous.
→ only use the constraints in air or interface between air and conductor

### Quadratic-nodal-linear-edge tetrahedral element

10 nodal DOFs

6 edge DOFs





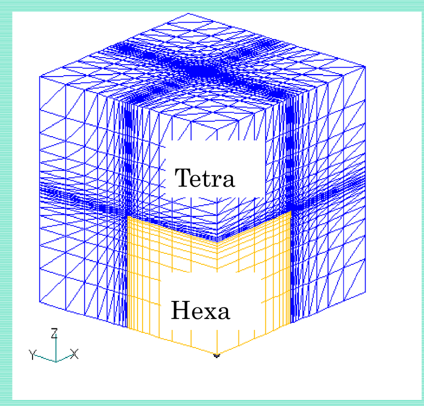
•Only addition of quadratic nodal functions to linear edge element.
→Remedy the degradation of linear tetrahedral elements

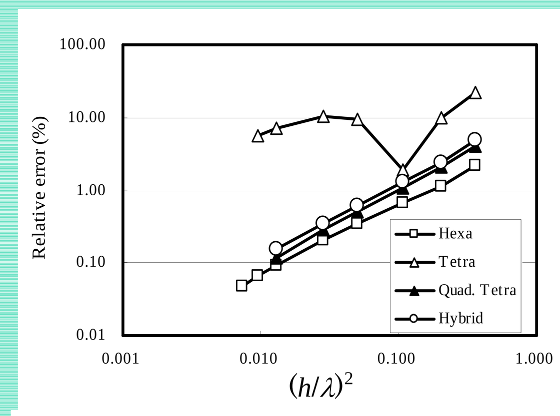
## Conclusion

◆Hybrid mesh of linear hexahedral and tetrahedral edge elements with non-conformal constraints makes mesh generation flexible and is effective in the eddy current analyses with thin skin depth in complicatedly shaped structures.

## Numerical Tests

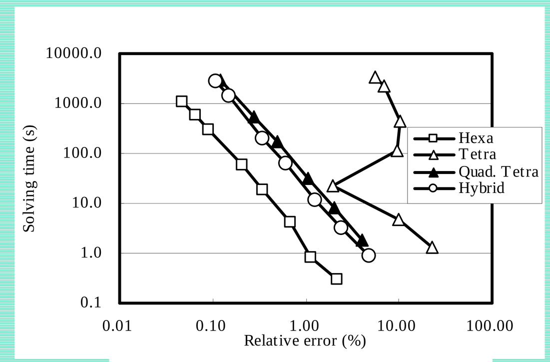
### *Cubic conductor*



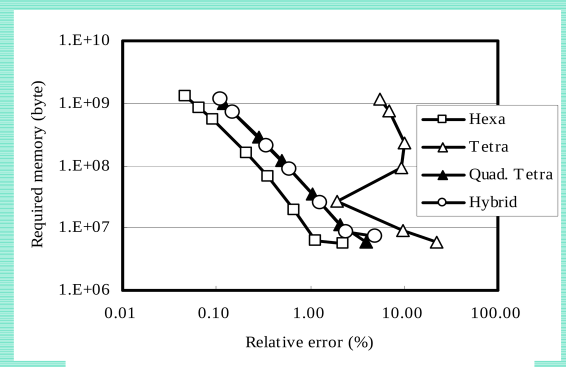




Error in heat loss v.s. mesh width *h*

◆Using Hybrid mesh, Errors are proportional to the square of mesh width in the same way as using other meshes.



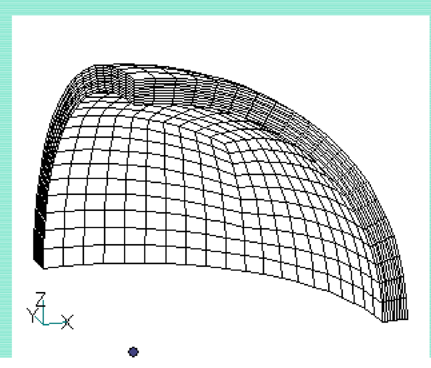

CPU time v.s. error



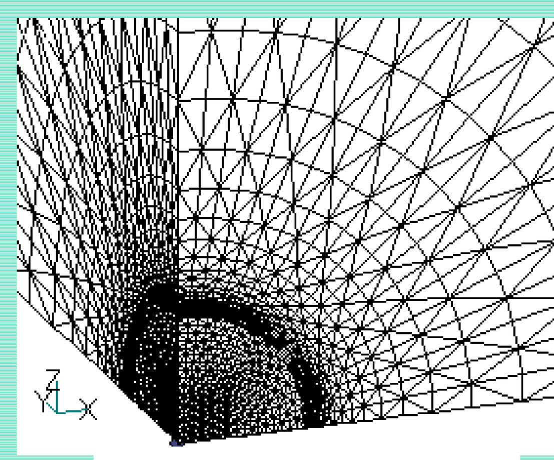

Required memory v.s. error

◆Hexahedral mesh is superior to the others.
◆Hybrid mesh is a little faster than Quad Tetra mesh.
◆Tetra mesh is too bad.

### *Hollow Sphere Model (TEAM Workshop Problem #6)*



Hexahedral mesh in conductor



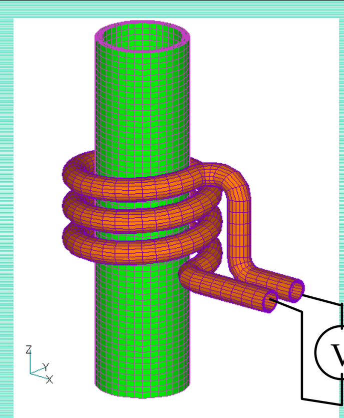
Tetrahedral mesh in air

Table I. Comparison between calculations using different meshes.

| | Hexa | Tetra | Quad-Tetra | Hybrid |
|---|---|---|---|---|
| No. Elements | 13,000 | 65,000 | 65,000 | 53,000 |
| No. Nodes | 14,571 | 17,490 | 37,330 | 14,571 |
| No. Unknowns | 41,000 | 79,400 | 97,990 | 70,200 |
| No. Nonzeros | 795,290 | 767,709 | 1,316,007 | 817,375 |
| ICCG Iterations | 69 | 169 | 169 | 114 |
| Solving Time (s) | 4.0 | 9.7 | 16.2 | 8.6 |
| Joule Heat (W) | 10,010 | 10,513 | 10,040 | 10,046 |
| Error (%) | 0.51 | 4.48 | 0.22 | 0.16 |

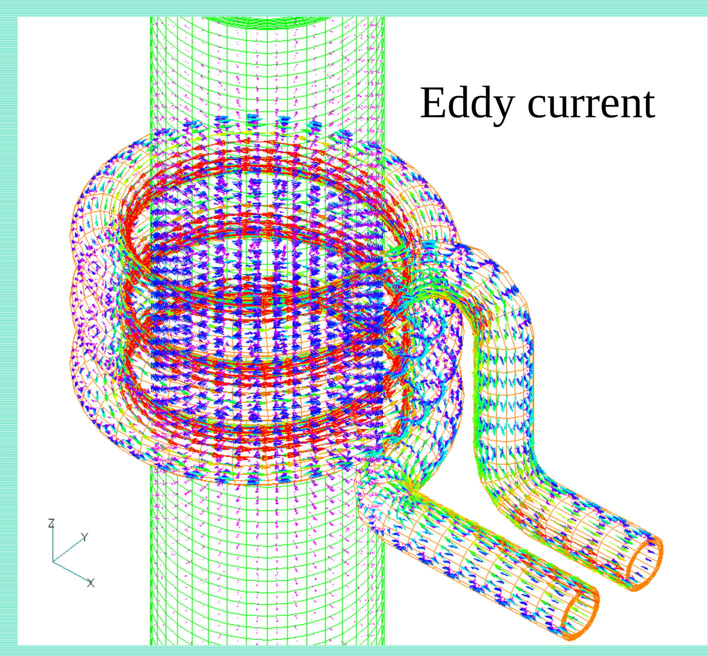
•**CPU time**
**Hexa < Hybrid< Tetra< Quad-Tetra**
•**Error**
**Hybrid < Quad-Tetra < Hexa << Tetra**

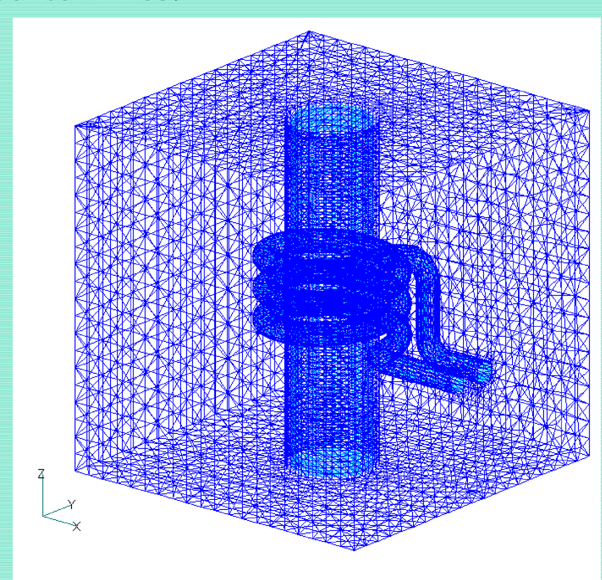
### *Model of Induction Heating Device using Hybrid mesh*



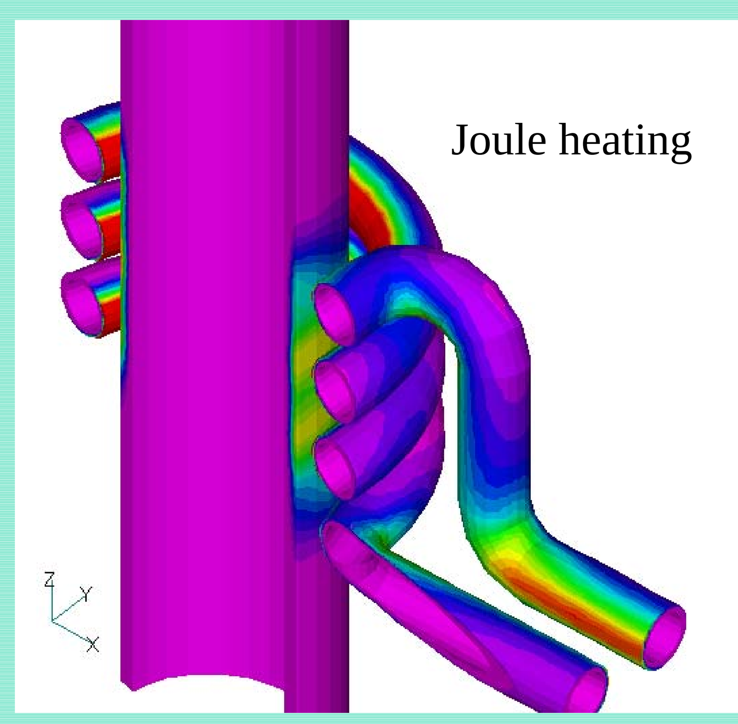

Hexahedral mesh of conductors

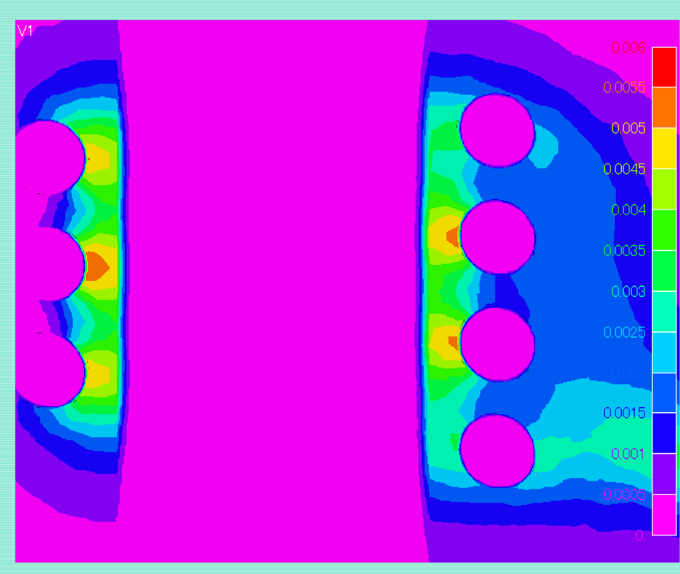
by sweeping 2-D mesh in the cross section along center lines.



Tetrahedral mesh of air region
by Delauney triangulation from surface triangular patch.



Magnetic field