

# Regularization on Ill-posed Source Terms in FEM Computation Using Two Magnetic Vector Potentials

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Purpose:

To solve the divergence of ICCG Method in the  
formulation using two magnetic vector potentials.

# Two Magnetic Vector Potentials

- Source currents are defined independently from FEM mesh.
- The mesh can be truncated relatively to the domains of magnetic material and/or conductors, not to source current.



- Easy in FEM discretization.
- The source current can be moved freely.
- **But, the ICCG method diverges easily.**

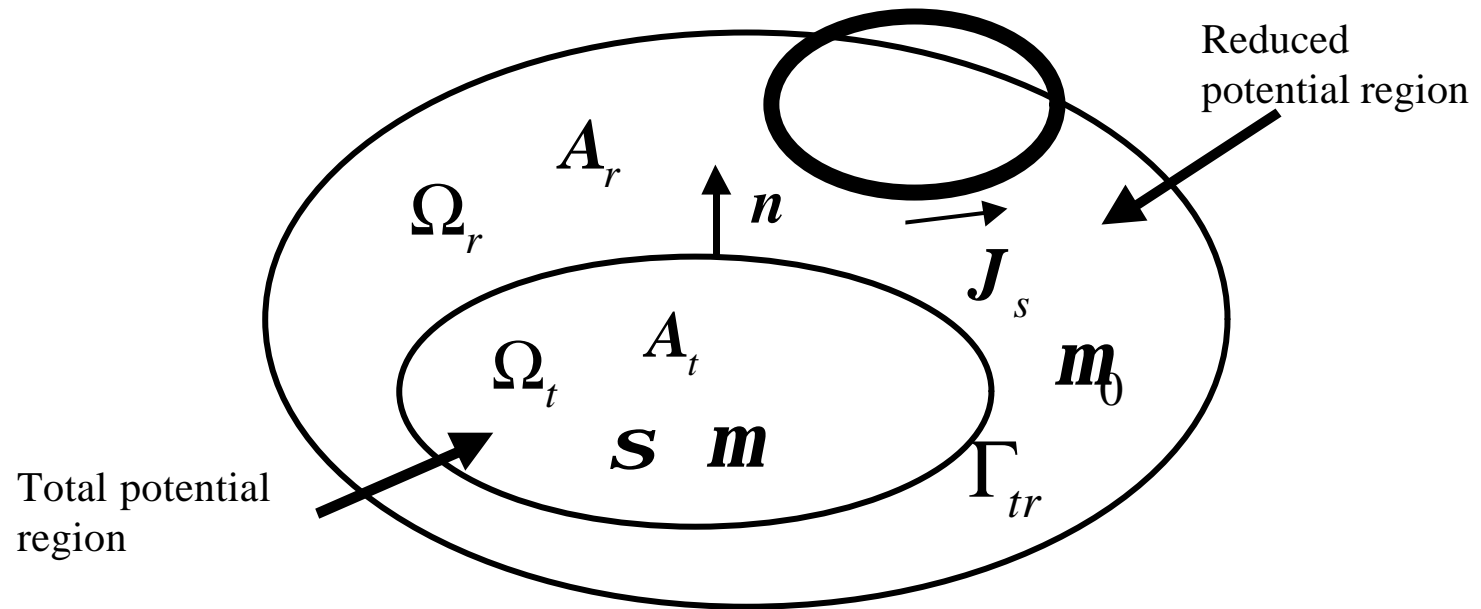


Fig. 1. Analysis regions for two potential method

- In total potential region ( $\Omega_t$ ),

$$\nabla \times \frac{1}{\mathbf{m}} \nabla \times \mathbf{A}_t + \mathbf{S} \frac{\partial \mathbf{A}_t}{\partial t} = 0$$

- In reduced potential region ( $\Omega_r$ ),  $\mathbf{A}_t = \mathbf{A}_r + \mathbf{A}_s$

$$\nabla \times \frac{1}{\mathbf{m}_0} \nabla \times \mathbf{A}_r = 0$$

$$\mathbf{A}_s = \frac{\mathbf{m}_0}{4\mathbf{p}} \int \frac{\mathbf{J}_s}{r} dV$$

- On the interface ( $\Gamma_{tr}$ ),

$$\mathbf{H}_t \times \mathbf{n} = (\mathbf{H}_r + \mathbf{H}_s) \times \mathbf{n}$$

$$\mathbf{H}_r = 1/\mathbf{m}_0 \nabla \times \mathbf{A}_r$$

$$\mathbf{H}_t = 1/\mathbf{m} \nabla \times \mathbf{A}_t$$

$$\mathbf{A}_t \times \mathbf{n} = (\mathbf{A}_r + \mathbf{A}_s) \times \mathbf{n}$$

$$\mathbf{H}_s = \frac{1}{4\mathbf{p}} \int \frac{\mathbf{J}_s \times \mathbf{r}}{r^3} dV$$

- Galerkin's weak form,

$$\int_{\Omega_t} \left( \nabla \times N \cdot \frac{1}{\mathbf{m}} \nabla \times \mathbf{A}_t + N \cdot \mathbf{S} \frac{\partial \mathbf{A}_t}{\partial t} \right) dV + \int_{\Omega_r} \left( \nabla \times N \cdot \frac{1}{\mathbf{m}_0} \nabla \times \mathbf{A}_r \right) dV$$

$$= \int_{\Gamma_{tr}} N \times \mathbf{H}_s \cdot \mathbf{n} dS.$$

- The source term of R.H.S must be in the range of the curl-curl operator.



- When the shape function  $N$  is substituted by  $\nabla f$  of a scalar potential in the allowable space of  $N$ , the source term must be zero.

$$\int_{\Gamma_{tr}} \nabla \mathbf{w} \times \mathbf{H}_s \cdot \mathbf{n} dS = \int_{\Gamma_{tr}} \mathbf{w} \nabla \times \mathbf{H}_s \cdot \mathbf{n} dS + \oint \mathbf{w} \mathbf{H}_s dC$$

■ Ideally, the source term is zero unless the current flows through the interface. But, numerically, integration errors are included and it doesn't become zero strictly.



■ The equation becomes inconsistent.



■ The ICCG method diverges.

# Example of the divergence of ICCG

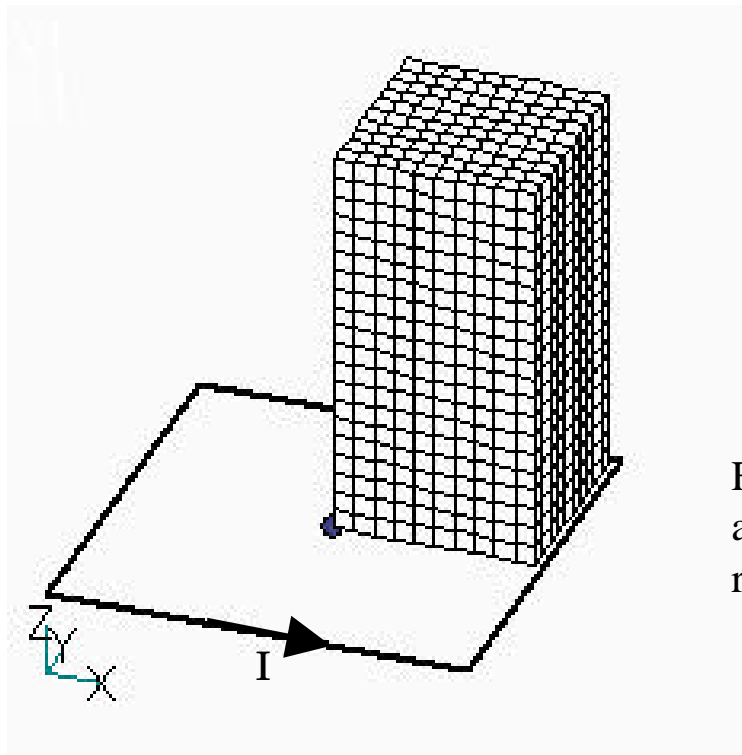


Fig.2 Test model with an iron core and a rectangular line current. Only 1/8 region is shown for the iron core.

■ In the case, an analytical solution is used for line current field. The accuracy of the integration depends only on the number of Gaussian integration points.

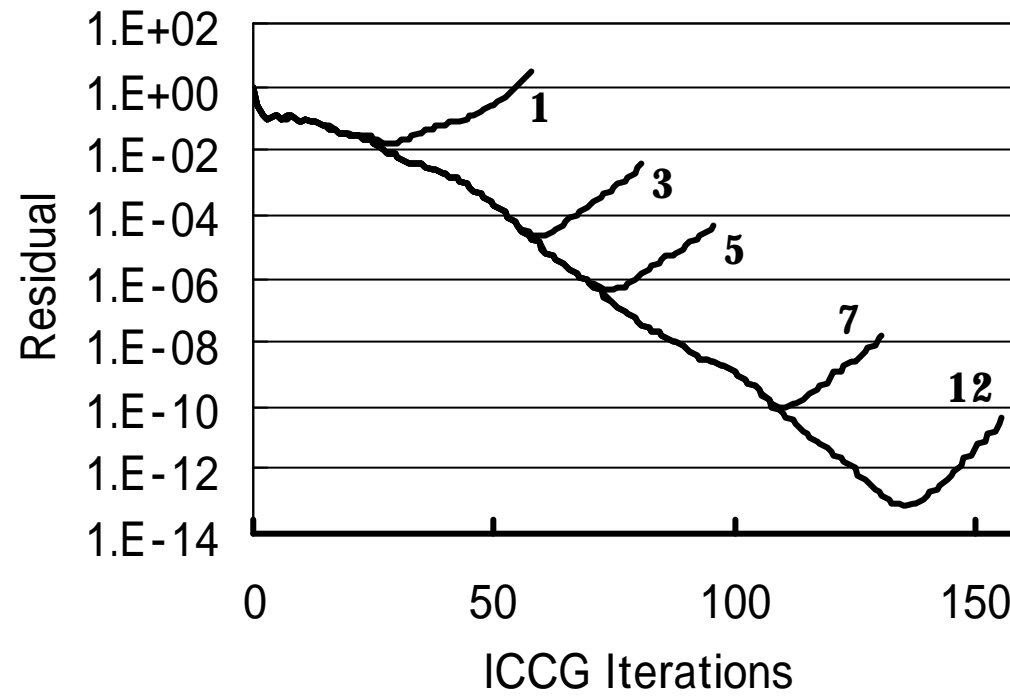


Fig. 3. Behavior in the ICCG iteration in the test problem. The numbers show the numbers of integration points in one direction.

- The ICCG converges to the level of the accuracy of the source term, after that, it diverges. One can approximate the solution by one at the minimum residual. The solution is including the error of the residual.
- We can attain more accurate solutions by increasing the accuracy of the source term.
- But it needs much time to calculate the integration.
- It makes the judgment for stopping the iteration complicated, because the convergence of ICCG is not monotonic in general.
- It is preferable and practical to get converged solution with less accurate source terms.

# Regularization of the source term

$$\tilde{\mathbf{H}}_s = \mathbf{H}_s - \nabla \mathbf{j} \times \mathbf{n} = \mathbf{H}_s - \nabla \times (\mathbf{j} \mathbf{n})$$

$$\begin{aligned} & \int_{\Gamma_{tr}} \nabla \mathbf{w} \times \tilde{\mathbf{H}}_s \cdot \mathbf{n} dS \\ &= \int_{\Gamma_{tr}} \nabla \mathbf{w} \times \mathbf{H}_s \cdot \mathbf{n} dS + \int_{\Gamma_{tr}} \nabla \mathbf{w} \times \mathbf{n} \cdot \nabla \mathbf{j} \times \mathbf{n} dS = 0 \end{aligned}$$

- The equation is positive-definite with unknowns on nodes in the interface and easily solved.

# Result of the regularization

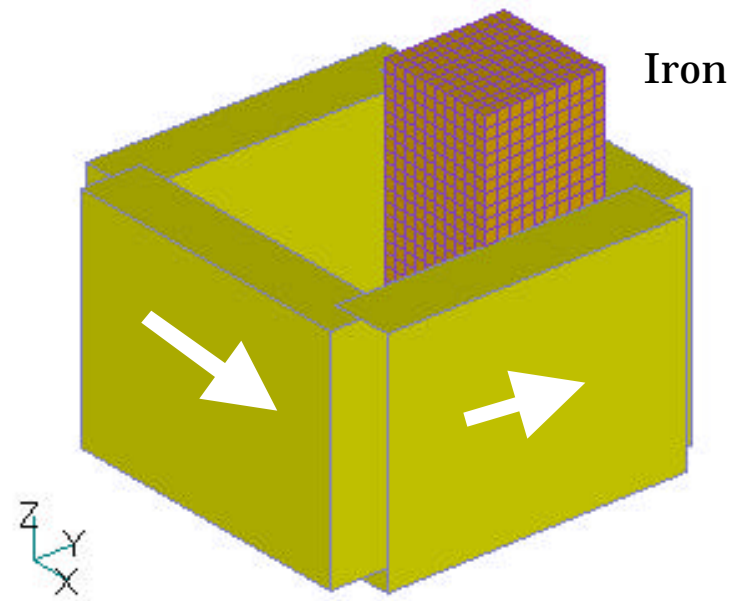


Fig. 4. Model for test calculation. The iron core is shown for 1/8 region.

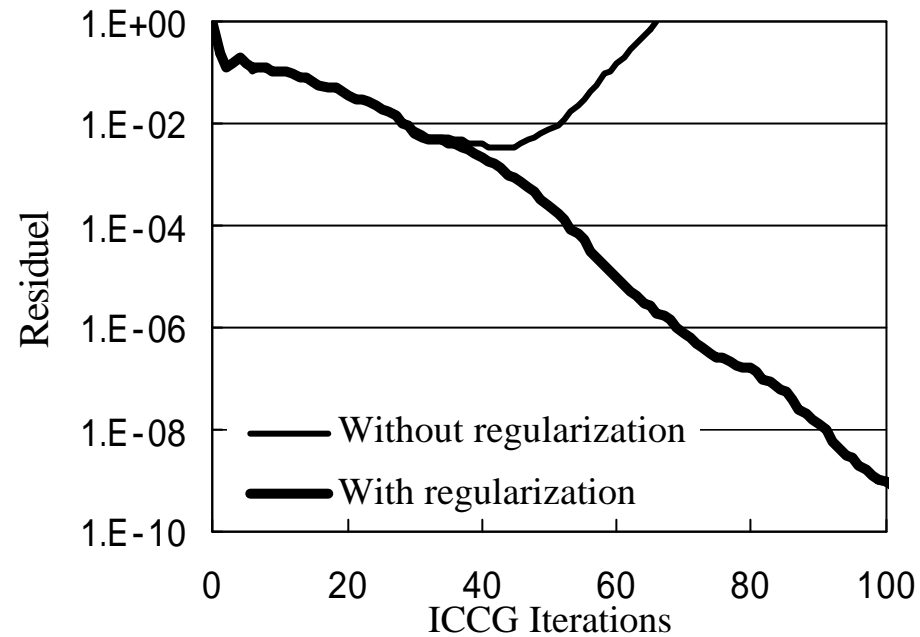
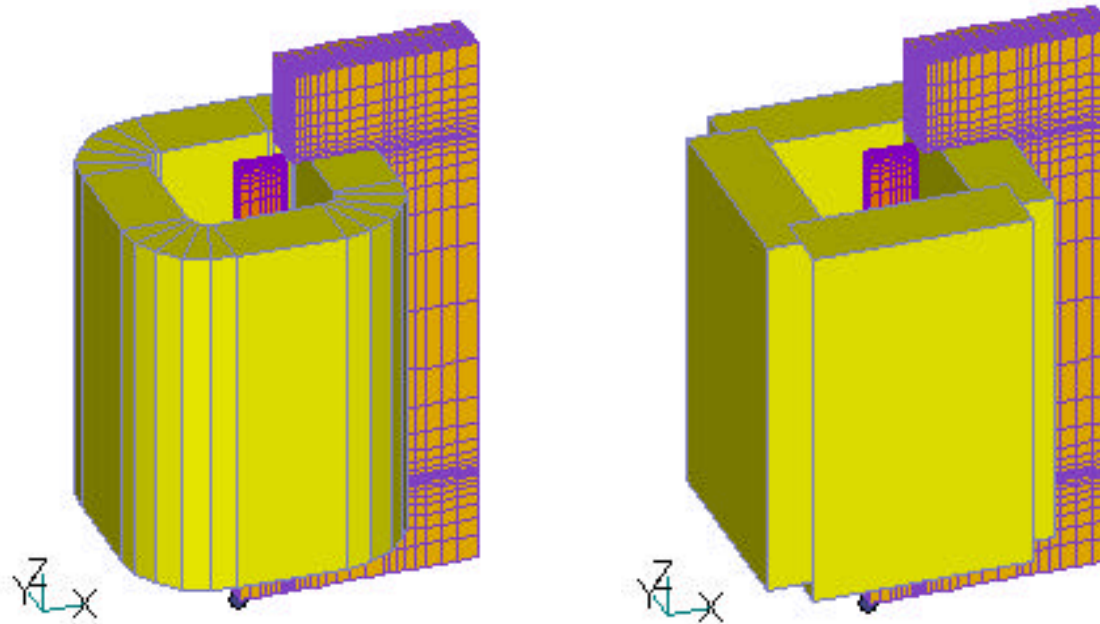


Fig. 5. Convergence of ICCG iterations with and without the regularization.

■ Even in the case where the current continuity is not satisfied in the model, the ICCG converges.



# Test of Team Workshop Problem #20

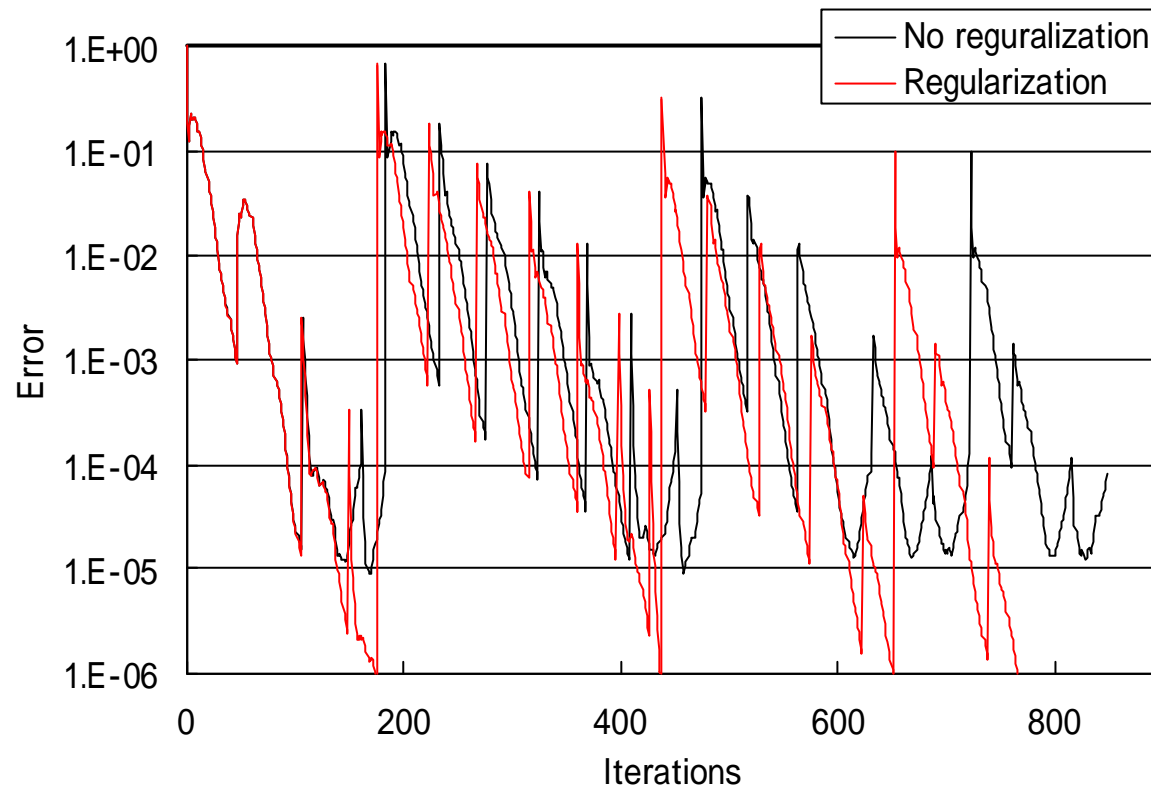


(a) Accurate model

(b) Rough model

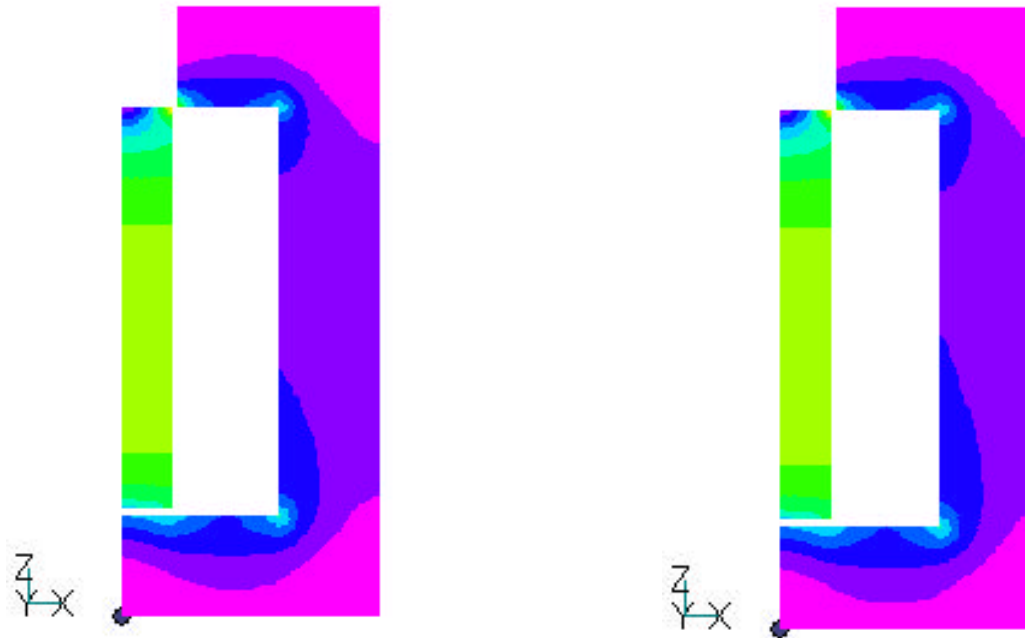
Fig.7 Models for TEAM Workshop Problem 20.

# Convergence of iterations with and without the regularization



Accurate model

# Regularized result



(a) Accurate model

(b) Rough model

Fig.8 Distribution of intensity of magnetic flux density in the result with the regularization

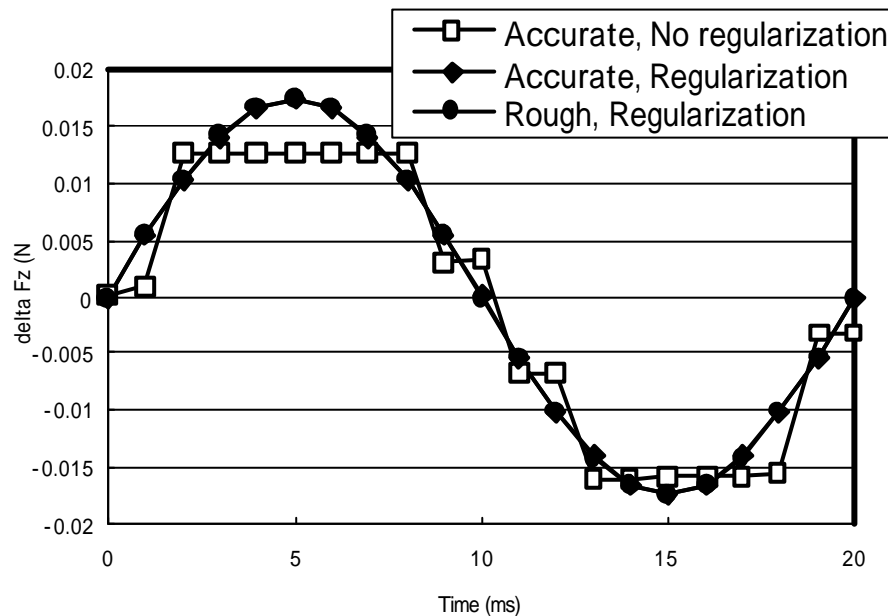
Table 1 Magnetic force at the center pole (N)

Model	Accurate	Accurate	Rough	Rough
Regularization	Off	On	Off	On
1000A	8.34448	8.34432	8.67932	8.62692
3000A	56.1728	56.1752	56.6812	57.0708
4500A	76.2668	76.2712	78.0924	77.1644
5000A	81.498	81.496	84.0876	82.4112

- In the accurate model, the difference is very little with and without the regularization.
- In the rough model, the result is nearer to the accurate model with regularization. Of course, it include the modeling error.

# Response to small disturbance

- Add 1A with sinusoidal waveform to 3000A current.



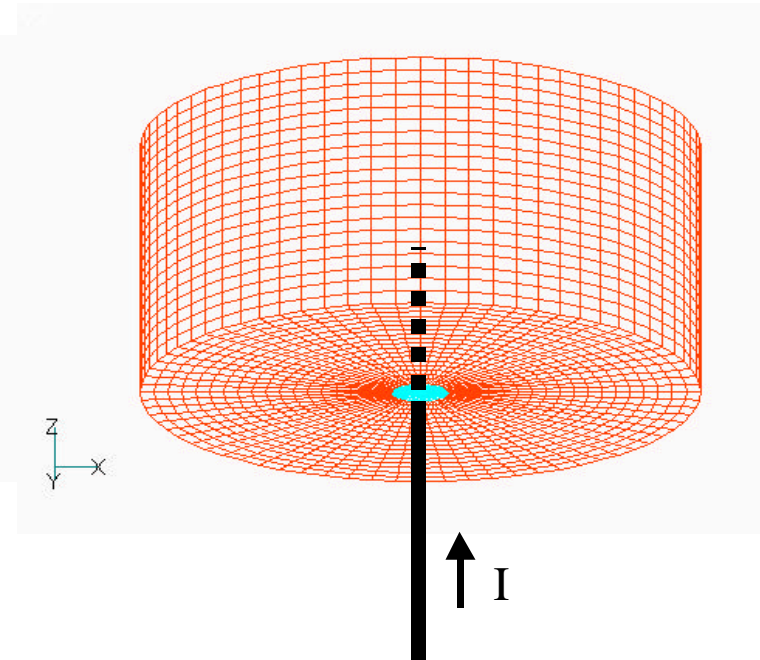
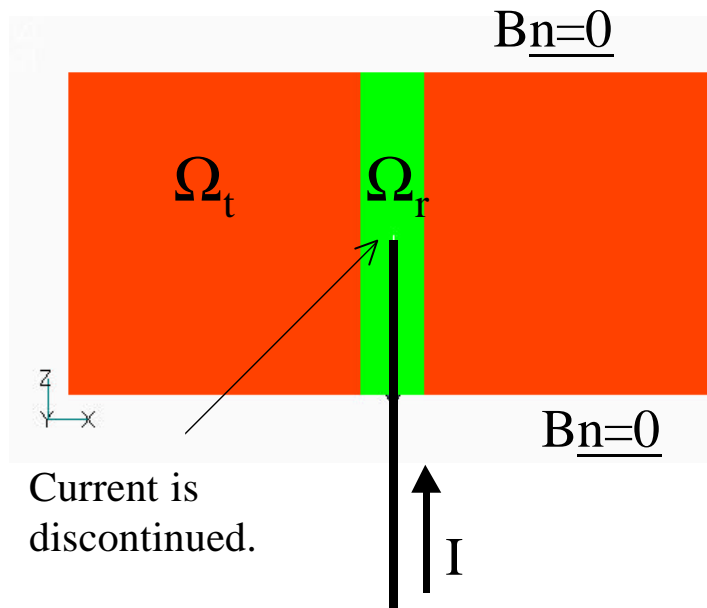
No reasonable result for rough model without regularization

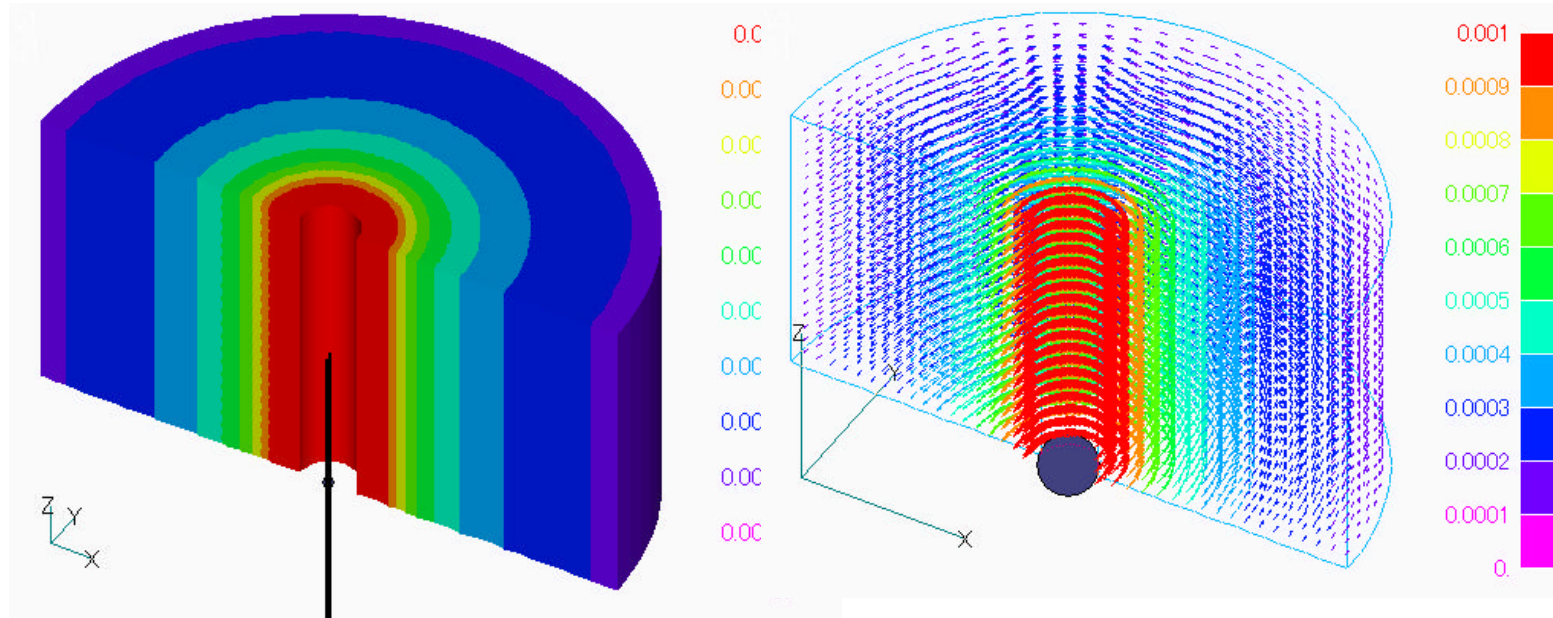
Zero levels of accurate and rough models are different.

Fig. 9 Response of the magnetic force on the center pole for 1A disturbance in the coil current

- Nonlinear iteration never converges under the level of ICCG convergence. With the regularization, because of the high convergence, one can calculate the response to a very small disturbance.
- The irregularity in the response would be fatal in transient calculations.

# What happens by the regularization?



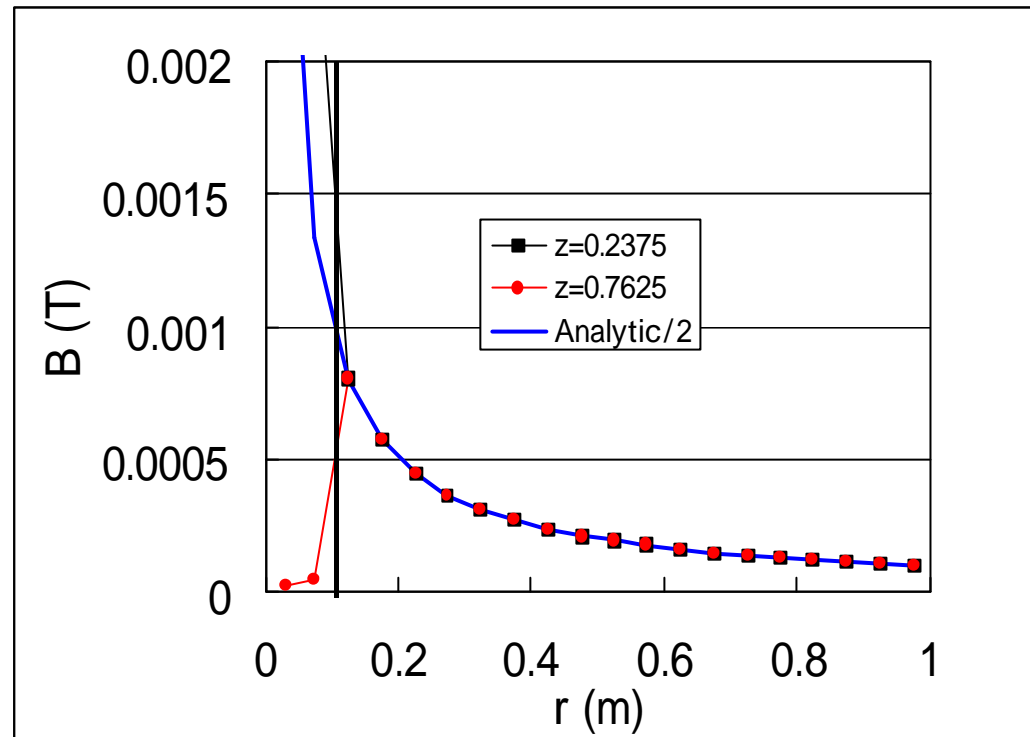


(a) Magnitude

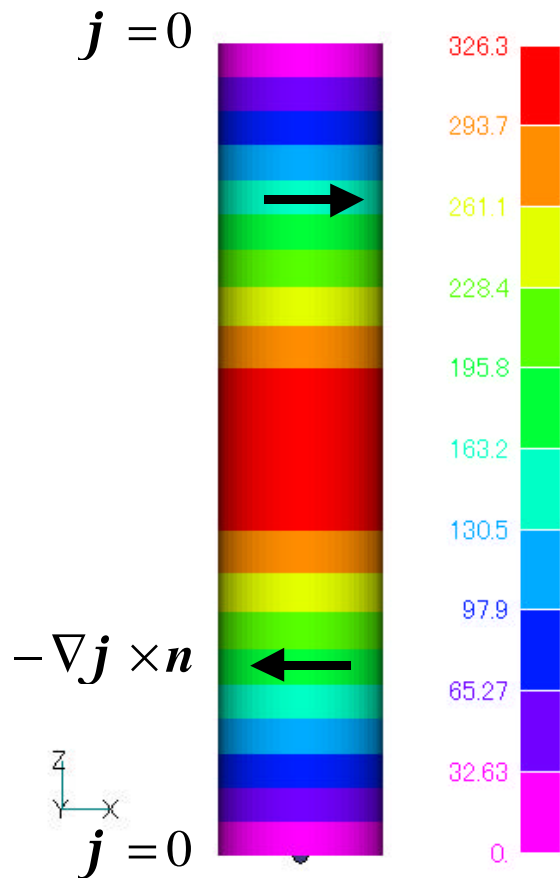
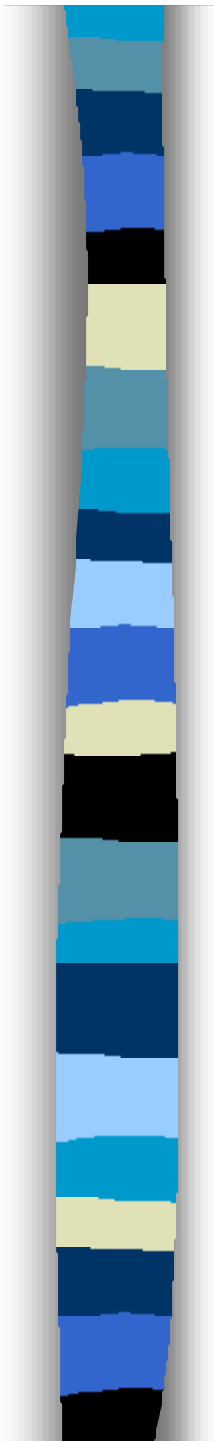
(b) Arrows

Magnetic flux density

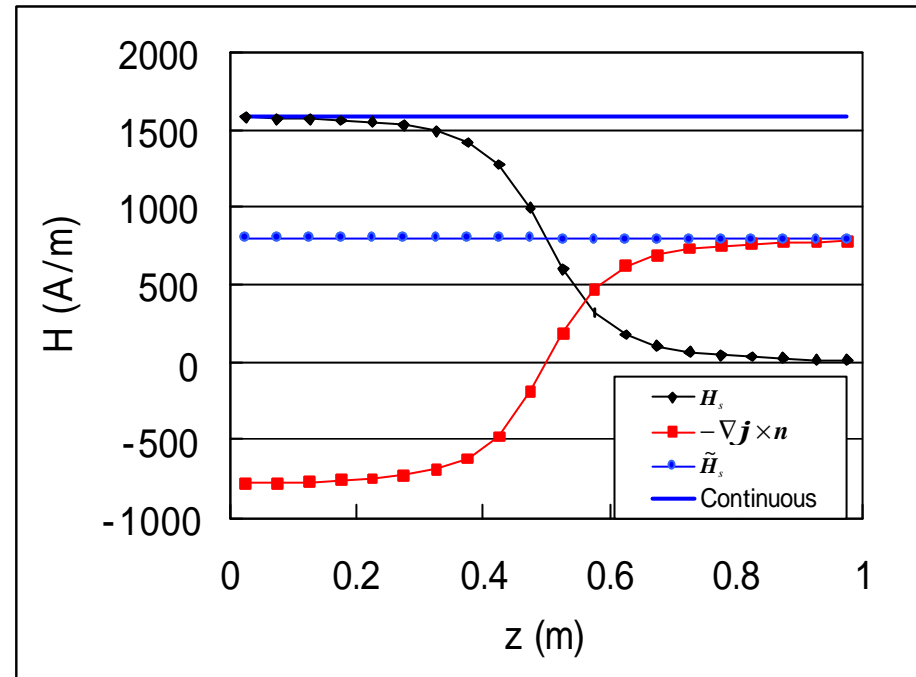
- No dependence on z direction in total potential region.



■ Averaged and half of continuous case.



Regularization potential



# Conclusion

- With the regularization, one can attain high convergence with less accurate source terms and even using fairly rough model.
- It makes the modeling easy and the integration of the source term shorter to calculate.
- It also make possible to calculate responses to smaller disturbances than the errors in source term.